1.5 期待値と分散

$\boxed{2}$ Xのとりうる値は 0, 1, 2. $P(X=0)=\left(\frac{1}{2}\right)^2$, $P(X=1)=\left(\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)^2$, $P(X=2)=\left(\frac{1}{2}\right)^2$

X	0	1	2	計
$P(X=k)$	1/4	1/2	1/4	1

$E(X)=0\times\frac{1}{4}+1\times\frac{1}{2}+2\times\frac{1}{4}=1$

$V(X)=E(X^2)-\{E(X)\}^2$

$=0^2\times\frac{1}{4}+1^2\times\frac{1}{2}+2^2\times\frac{1}{4}-1=\frac{1}{2}$

$\sigma(X)=\sqrt{V(X)}=\frac{1}{\sqrt{2}}=\frac{\sqrt{2}}{2}$

$\boxed{3}$ Xのとりうる値は. 0, 1, 2, 3.

$P(X=0)=\frac{{}_6C_3}{{}_{10}C_3}=\frac{20}{120}=\frac{1}{6}$, $P(X=1)=\frac{{}_4C_1\cdot{}_6C_2}{{}_{10}C_3}=\frac{4\times15}{120}=\frac{1}{2}$

$P(X=2)=\frac{{}_4C_2\cdot{}_6C_1}{{}_{10}C_3}=\frac{6\times6}{120}=\frac{3}{10}$, $P(X=3)=\frac{{}_4C_3}{{}_{10}C_3}=\frac{4}{120}=\frac{1}{30}$

X	0	1	2	3	計
$P(X=k)$	1/6	1/2	3/10	1/30	1

$E(X)=0\times\frac{1}{6}+1\times\frac{1}{2}+2\times\frac{3}{10}+3\times\frac{1}{30}=\frac{6}{5}$

$V(X)=E(X^2)-\{E(X)\}^2=0^2\times\frac{1}{6}+1^2\times\frac{1}{2}+2^2\times\frac{3}{10}+3^2\times\frac{1}{30}-\left(\frac{6}{5}\right)^2=\frac{14}{25}$

$\sigma(X)=\sqrt{V(X)}=\frac{\sqrt{14}}{5}$

$\boxed{4}$ $E(X)=\int_{-\infty}^{\infty}x\,p(x)dx=\int_0^2 x\left(\frac{1}{3}+\frac{1}{6}x\right)dx=\int_0^2\left(\frac{1}{3}x+\frac{1}{6}x^2\right)dx=\left[\frac{1}{6}x^2+\frac{1}{18}x^3\right]_0^2$

$=\frac{4}{6}+\frac{8}{18}=\frac{10}{9}$

$V(X)=E(X^2)-\{E(X)\}^2=\int_0^2 x^2\left(\frac{1}{3}+\frac{1}{6}x\right)dx-\left(\frac{10}{9}\right)^2=\int_0^2\left(\frac{1}{3}x^2+\frac{1}{6}x^3\right)dx-\frac{100}{81}$

$=\left[\frac{1}{9}x^3+\frac{1}{24}x^4\right]_0^2-\frac{100}{81}=\frac{8}{9}+\frac{16}{24}-\frac{100}{81}=\frac{26}{81}$ $\therefore\ \sigma(X)=\sqrt{V(X)}=\frac{\sqrt{26}}{9}$

$\boxed{5}$ $E(X)=\int_0^{\infty}x\cdot2e^{-2x}dx=2\int_0^{\infty}xe^{-2x}dx$

$I=\int_0^{\infty}xe^{-2x}dx=\left[-\frac{1}{2}xe^{-2x}\right]_0^{\infty}-\int_0^{\infty}\left(-\frac{1}{2}e^{-2x}\right)dx=-\frac{1}{2}\overbrace{\lim_{x\to\infty}\frac{x}{e^{2x}}}^{=0}+\frac{1}{2}\int_0^{\infty}e^{-2x}dx$

$=\frac{1}{2}\left[-\frac{1}{2}e^{-2x}\right]_0^{\infty}=-\frac{1}{4}\Big(\underbrace{\lim_{x\to\infty}\frac{1}{e^{2x}}}_{=0}-1\Big)=\frac{1}{4}$ $\therefore\ E(X)=\frac{1}{2}$

$$V(X)=E(X^2)-\{E(X)\}^2=\int_0^\infty x^2\cdot 2e^{-2x}dx-\left(\frac{1}{2}\right)^2=2\int_0^\infty x^2e^{-2x}dx-\frac{1}{4}$$

$$\int_0^\infty x^2\cdot e^{-2x}dx=\left[x^2\left(-\frac{1}{2}e^{-2x}\right)\right]_0^\infty-\int_0^\infty 2x\left(-\frac{1}{2}e^{-2x}\right)dx=-\frac{1}{2}\overbrace{\lim_{x\to\infty}\frac{x^2}{e^{2x}}}^{=0}+\int_0^\infty xe^{-2x}dx$$

$$=\frac{1}{4}\quad\therefore V(X)=2\times\frac{1}{4}-\frac{1}{4}=\frac{1}{2}-\frac{1}{4}=\frac{1}{4}$$

$\boxed{6}$ 連続型の場合に示す: $E((X-a)^2)=\int_{-\infty}^{\infty}(x-a)^2p(x)dx=\int_{-\infty}^{\infty}(x^2-2ax+a^2)p(x)dx$

$$=\int_{-\infty}^{\infty}x^2p(x)dx-2a\int_{-\infty}^{\infty}xp(x)dx+a^2\int_{-\infty}^{\infty}p(x)dx=E(X^2)-2aE(X)+a^2$$

$$=V(X)+\{E(X)\}^2-2aE(X)+a^2=V(X)+\{E(X)-a\}^2$$

$\boxed{7}$ $E(Y)=aE(X)+b$ なので $(Y-E(Y))^k=\{(aX+b)-(aE(X)+b)\}^2=a^k\cdot(X-E(X))^k$

$$\therefore E[(Y-E(Y))^k]=a^kE[(X-E(X))^k].$$