1.4 連続型確率変数と確率密度関数

$\boxed{2}$ (1) $P(-2 \leqq X \leqq -1) = \int_{-2}^{-1}\left(\frac{1}{4}+\frac{1}{8}x\right)dx = \frac{1}{4}(-1-(-2)) + \frac{1}{16}\left[x^2\right]_{-2}^{-1} = \frac{1}{4}+\frac{1}{16}(1-4)$

$= \frac{4}{16} - \frac{3}{16} = \frac{1}{16}$

(2) $P(X \geqq 1) = \int_{1}^{2}\left(\frac{1}{4}+\frac{1}{8}x\right)dx = \frac{1}{4}(2-1) + \frac{1}{16}\left[x^2\right]_{1}^{2} = \frac{1}{4}+\frac{1}{16}(4-1) = \frac{4}{16}+\frac{3}{16} = \frac{7}{16}$

(3) ・$x<-2$の時：$F(x) = \int_{-\infty}^{x} p(t)dt = 0$

・$-2 \leqq x \leqq 2$の時：$F(x) = \int_{-\infty}^{-2} p(t)dt + \int_{-2}^{x} p(t)dt = \int_{-2}^{x}\left(\frac{1}{4}+\frac{1}{8}t\right)dt$

$= \frac{1}{4}(x+2) + \frac{1}{16}\left[t^2\right]_{-2}^{x} = \frac{1}{4}x + \frac{1}{2} + \frac{1}{16}(x^2-4) = \frac{1}{4}x + \frac{1}{16}x^2 + \frac{1}{4}$

・$x>2$の時：$F(x) = \int_{-\infty}^{-2} p(t)dt + \int_{-2}^{2} p(t)dt + \int_{2}^{x} p(t)dt = \int_{-2}^{2}\left(\frac{1}{4}+\frac{1}{8}t\right)dt$

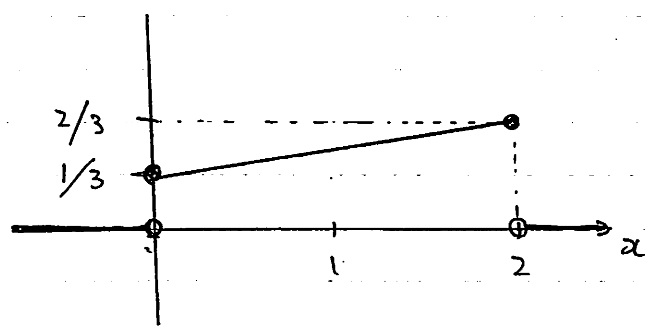
$= \frac{1}{4}(2-(-2)) + \frac{1}{16}\left[t^2\right]_{-2}^{2} = 1$

以上より

$$F(x) = \begin{cases} 0 & (x<-2) \\ \frac{1}{4}x + \frac{1}{16}x^2 + \frac{1}{4} & (-2 \leqq x \leqq 2) \\ 1 & (x>2) \end{cases}$$

$\boxed{3}$ $1 = \int_{-\infty}^{\infty} p(x)dx = \int_{0}^{2}\left(\frac{1}{3}+ax\right)dx = \frac{2}{3} + \frac{a}{2}\left[x^2\right]_{0}^{2} = \frac{2}{3}+2a$. $\therefore 2a = \frac{1}{3}$ $\therefore a = \frac{1}{6}$

$$\therefore p(x) = \begin{cases} \frac{1}{3} + \frac{1}{6}x & (0 \leqq x \leqq 2) \\ 0 & (その他) \end{cases}$$



次に $F(x)$ を求める：

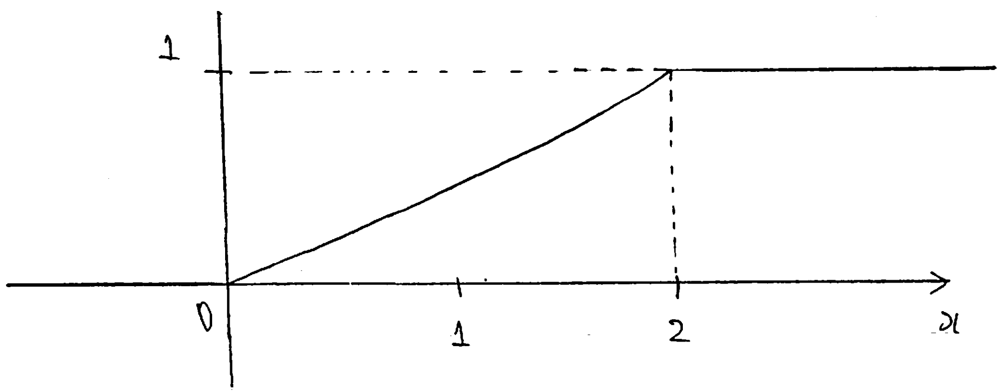
・$x<0$のとき：$F(x) = \int_{-\infty}^{x} p(t)dt = 0$

・$0 \leqq x \leqq 2$のとき：$F(x) = \int_{-\infty}^{0} p(t)dt + \int_{0}^{x} p(t)dt = \int_{0}^{x}\left(\frac{1}{3}+\frac{1}{6}t\right)dt = \frac{x}{3} + \frac{1}{12}x^2$

• $x>2$ のとき: $F(x)=\int_{-\infty}^{0}p(t)dt+\int_{0}^{2}p(t)dt+\int_{2}^{x}p(t)dt=\int_{0}^{2}\left(\frac{1}{3}+\frac{1}{6}t\right)dt=1$

以上より

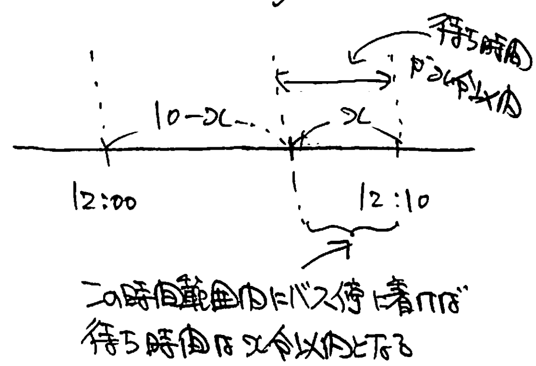
$$F(x)=\begin{cases}0 & (x<0)\\ \frac{x}{3}+\frac{1}{12}x^2 & (0\leqq x\leqq 2)\\ 1 & (x>2)\end{cases}$$



$\boxed{4}$ 分布関数 $F(x):=P(X\leqq x)$ (= 待ち時間がx分以内である確率) を求める.

• $x<0$ の時: $F(x)=P(X\leqq x)=P(\phi)=0$

• $0\leqq x\leqq 10$ の時: $F(x)=P(X\leqq x)=\frac{x}{10}$



• $x>10$ の時: $F(x)=P(X\leqq x)=P(\Omega)=1$

$$\therefore F(x)=\begin{cases}0 & (x<0)\\ \frac{1}{10}x & (0\leqq x\leqq 10)\\ 1 & (x>10)\end{cases}\qquad \therefore p(x)=\frac{dF(x)}{dx}=\begin{cases}\frac{1}{10} & (0\leqq x\leqq 10)\\ 0 & (x<0,\ x>10)\end{cases}$$

X= 待ち時間(分)とすると, $0\leqq X\leqq 10$.