解答例

1. (1) $f(x)$ は奇 より $a_n = 0$.

$$b_n = \frac{2}{\pi}\int_0^{\pi} x \sin nx \, dx = \frac{2}{\pi}\left[-\frac{1}{n} x \cos nx\right]_0^{\pi} + \frac{2}{\pi n}\int_0^{\pi} \cos nx \, dx$$

$$= -\frac{2}{n}(-1)^n + \frac{2}{\pi n}\left[\frac{1}{n}\sin nx\right]_0^{\pi} = \frac{2}{n}(-1)^{n+1}.$$

$$\therefore f(x) \sim \sum_{n=1}^{\infty} \frac{2}{n}(-1)^{n+1} \cdot \sin nx.$$

(2). $$\frac{1}{\pi}\int_{-\pi}^{\pi} x^2 \, dx = \frac{1}{\pi}\left[\frac{1}{3}x^3\right]_{-\pi}^{\pi} = \frac{2}{3}\pi^2.$$

$$\sum_{n=1}^{\infty} b_n^2 = \sum_{n=1}^{\infty}\left(\frac{2}{n}\right)^2 = \sum_{n=1}^{\infty}\frac{4}{n^2}$$

$$\therefore \frac{2}{3}\pi^2 = \sum_{n=1}^{\infty}\frac{4}{n^2}$$ より. $$\sum_{n=1}^{\infty}\frac{1}{n^2} = \frac{1}{6}\pi^2.$$

2. $$S(u) = \sqrt{\frac{2}{\pi}}\int_0^{a} \sin ut \, dt = \sqrt{\frac{2}{\pi}}\left[-\frac{1}{u}\cos ut\right]_0^{a}$$

$$= \sqrt{\frac{2}{\pi}} \cdot \frac{1}{u}(1 - \cos au)$$

3. $$\hat{f}(u) = \frac{1}{\sqrt{2\pi}}\int_{-1}^{1} |t| \cdot e^{-iut} \, dt$$

$$= \frac{1}{\sqrt{2\pi}}\left(\int_0^1 t e^{-iut} \, dt - \int_{-1}^{0} t e^{-iut} \, dt\right) =$$

$$= \frac{1}{\sqrt{2\pi}} \Bigg(\left[-\frac{1}{iu} t e^{-iut} \right]_0^1 - \frac{i}{u} \int_0^1 e^{-iut} dt$$

$$- \left[-\frac{1}{iu} t e^{-iut} \right]_{-1}^0 + \frac{i}{u} \int_{-1}^0 e^{-iut} dt \Bigg).$$

$$= \frac{1}{\sqrt{2\pi}} \left(\frac{i}{u} e^{-iu} - \frac{i}{u} \left[-\frac{1}{iu} e^{-iut} \right]_0^1 - \frac{i}{u} e^{iu} + \frac{i}{u} \left[-\frac{1}{iu} e^{-iut} \right]_{-1}^0 \right).$$

$$= \frac{1}{\sqrt{2\pi}} \left(\frac{i}{u} (e^{-iu} - e^{iu}) + \frac{1}{u^2} (e^{-iu} - 1 - 1 + e^{iu}) \right)$$

$$= \frac{1}{\sqrt{2\pi}} \left(\frac{i}{u} (e^{-iu} - e^{iu}) + \frac{1}{u^2} (e^{-iu} + e^{iu} - 2) \right)$$

4. $\mathbf{a} \times \mathbf{b} = (2\cdot3 - 1\cdot1, 1\cdot2 - 3\cdot1, 1\cdot1 - 2\cdot2) = (5, -1, -3)$

$$|\mathbf{a}\ \mathbf{b}\ \mathbf{c}| = \begin{vmatrix} 1 & 2 & 1 \\ 2 & 1 & 3 \\ 2 & 0 & 1 \end{vmatrix} = 1 + 12 - 2 - 4 = 7.$$

5 $\dot{\mathbf{r}}(t) = (-\sqrt{2} \sin t, \sqrt{2} \cos t, \sqrt{2}).$

$$|\dot{\mathbf{r}}(t)| = \sqrt{(-\sqrt{2} \sin t)^2 + (\sqrt{2} \cos t)^2 + 2} = 2.$$

$$\therefore S(t) = \int_0^t 2 \, d\tau = [2\tau]_0^t = 2t.$$

これより. $t = \frac{S}{2}$ なので. $\mathbf{r}(S) = \left(\sqrt{2} \cos\frac{S}{2}, \sqrt{2} \sin\frac{S}{2}, \frac{S}{\sqrt{2}}\right)$ である

(2) $\mathbf{e}_1(s) = \mathbf{r}'(s) = \left(-\frac{1}{\sqrt{2}}\sin\frac{s}{2}, \frac{1}{\sqrt{2}}\cos\frac{s}{2}, \frac{1}{\sqrt{2}}\right)$

$\mathbf{r}''(s) = \left(-\frac{1}{2\sqrt{2}}\cos\frac{s}{2}, -\frac{1}{2\sqrt{2}}\sin\frac{s}{2}, 0\right)$.

$\kappa(s) = |\mathbf{r}''(s)| = \sqrt{\frac{1}{8}\cos^2\frac{s}{2} + \frac{1}{8}\sin^2\frac{s}{2}} = \frac{1}{2\sqrt{2}}$

$\mathbf{e}_2(s) = \dfrac{\mathbf{r}''(s)}{\kappa(s)} = \left(-\cos\frac{s}{2}, -\sin\frac{s}{2}, 0\right)$

$\mathbf{e}_3(s) = \mathbf{e}_1(s) \times \mathbf{e}_2(s) = \left(\frac{1}{\sqrt{2}}\sin\frac{s}{2}, -\frac{1}{\sqrt{2}}\cos\frac{s}{2}, \frac{1}{\sqrt{2}}\right)$ である.

6. $\frac{\partial}{\partial u}\mathbf{r} = (\cos u\cos v, \cos u\sin v, -\sin u)$

$\frac{\partial}{\partial v}\mathbf{r} = (-\sin u\sin v, \sin u\cos v, 0)$ より

$\mathbf{r}_u \times \mathbf{r}_v = (\cos v\sin^2 u, \sin^2 u\sin v, \sin u\cos u)$

$|\mathbf{r}_u \times \mathbf{r}_v| = \sin u\sqrt{\cos^2 v\sin^2 u + \sin^2 v\sin^2 u + \cos^2 u} = \sin u$.

$\therefore S = \int_0^{2\pi}\int_0^{\pi}\sin u\,du\,dv = \int_0^{2\pi}\left[-\cos u\right]_0^{\pi}dv$

$= \int_0^{2\pi} 2\,dv = 4\pi$. である

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7. $\mathbf{r}(t) = (x(t), y(t))$ とすると $\dot{\mathbf{r}}(t) = (\dot{x}(t), \dot{y}(t))$ である.

$\dot{\mathbf{r}}(t) = \mathbf{a}(\mathbf{r}(t))$ より $\begin{cases} \dot{x}(t) = x(t) \\ \dot{y}(t) = 0 \end{cases}$ となる

$\dot{y}(t) = 0$ を解くと. $y(t) = C_2$.

$\dot{x}(t) = x(t)$ を解くと. $\frac{\dot{x}}{x} = 1$ より $\log x = t + C_1$

$x(t) = e^{t+C_1} = C_1 \cdot e^t$ (e^{C_1} を C_1 でおきかえた).

$\therefore$ $\mathbf{x}(t) = (C_1 e^t, C_2)$ である //.