解答例.

1. $f(x)$ は奇 なので. $a_n = 0$.

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx = \frac{2}{\pi}\int_0^{\pi} f(x)\sin nx\,dx$$

$$= \frac{2}{\pi}\int_0^{\pi}\sin nx\,dx = \frac{2}{\pi}\left[-\frac{1}{n}\cos nx\right]_0^{\pi}$$

$$= \frac{-2}{\pi n}\left((-1)^n - 1\right) = \frac{2}{\pi n}\left(1-(-1)^n\right)$$

$$\therefore f(x) \sim \sum_{n=1}^{\infty}\frac{2}{\pi n}\left(1-(-1)^n\right)\sin nx.$$

2. $$c_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi}|x|\,dx = \frac{1}{\pi}\int_0^{\pi}x\,dx = \frac{1}{\pi}\left[\frac{1}{2}x^2\right]_0^{\pi} = \frac{\pi}{2}.$$

$n \neq 0$ のとき.

$$c_n = \frac{1}{2\pi}\int_{-\pi}^{\pi}|x|\,e^{-inx}dx$$

$$= \frac{1}{2\pi}\int_{-\pi}^{0} -x\,e^{-inx}dx + \frac{1}{2\pi}\int_0^{\pi}x\,e^{-inx}dx$$

$$= \frac{1}{2\pi}\left(\left[\frac{1}{in}\cdot x\,e^{-inx}\right]_{-\pi}^{0} - \frac{1}{in}\int_{-\pi}^{0}e^{-inx}\right.$$

$$\left. - \left[\frac{1}{in}x\,e^{-inx}\right]_0^{\pi} + \frac{1}{in}\int_0^{\pi}e^{-inx}dx\right)$$

$$= \frac{1}{2\pi}\left(-\frac{i}{n}(-1)^n - \frac{1}{in}\left[-\frac{1}{in}e^{-inx}\right]_{-\pi}^{0} + \frac{i}{n}(-1)^n + \frac{1}{in}\left[-\frac{1}{in}e^{-inx}\right]_0^{\pi}\right)$$

$$= \frac{1}{2\pi}\left(-\frac{1}{n^2}(1-(-1)^n) + \frac{1}{n^2}((-1)^n-1)\right)$$

$$= \frac{1}{\pi n^2}((-1)^n-1)$$

$$\therefore f(x) \sim \frac{\pi}{2} + \sum_{n\neq 0} \frac{1}{\pi n^2}((-1)^n-1)e^{inx}$$

3. $$\hat{f}(u) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(t)e^{-iut}\,dt$$

$$= \frac{1}{\sqrt{2\pi}}\int_0^1 t e^{-iut}\,dt$$

$$= \frac{1}{\sqrt{2\pi}}\left(\left[-\frac{1}{iu}te^{-iut}\right]_0^1 + \frac{1}{iu}\int_0^1 e^{-iut}\,dt\right)$$

$$= \frac{1}{\sqrt{2\pi}}\left(\frac{i}{u}e^{-iu} + \frac{1}{iu}\left[-\frac{1}{iu}e^{-iut}\right]_0^1\right)$$

$$= \frac{1}{\sqrt{2\pi}}\left(\frac{i}{u}e^{-iu} + \frac{1}{u^2}(e^{-iu}-1)\right)$$

4. $a \times b = (6-1, 2-3, 1-4) = (5, -1, -3)$

$$|a\ b\ c| = \begin{vmatrix} 1 & 2 & 1 \\ 2 & 1 & 3 \\ 2 & 0 & 1 \end{vmatrix} = 1 + 12 - 2 - 4 = 7.$$

5. (1) $\dot{\mathbb{r}}(t) = (-\sin t, \cos t, 1)$

$|\dot{\mathbb{r}}(t)| = \sqrt{(-\sin t)^2 + \cos^2 t + 1^2} = \sqrt{2}$.

$\therefore S(t) = \int_0^t |\dot{\mathbb{r}}(\tau)| d\tau = \int_0^t \sqrt{2}\, d\tau = \sqrt{2}t$.

(2) $\mathbb{r}(s) = \left(\cos\frac{s}{\sqrt{2}}, \sin\frac{s}{\sqrt{2}}, \frac{s}{\sqrt{2}}\right)$ より

$\mathbb{e}_1(s) = \mathbb{r}'(s) = \left(-\frac{1}{\sqrt{2}}\sin\frac{s}{\sqrt{2}}, \frac{1}{\sqrt{2}}\cos\frac{s}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$.

$\mathbb{r}''(s) = \left(-\frac{1}{2}\cos\frac{s}{\sqrt{2}}, -\frac{1}{2}\sin\frac{s}{\sqrt{2}}, 0\right)$.

$\kappa(s) = |\mathbb{r}''(s)| = \sqrt{\frac{1}{4}\cos^2\frac{s}{\sqrt{2}} + \frac{1}{4}\sin^2\frac{s}{\sqrt{2}}} = \frac{1}{2}$

$\mathbb{e}_2(s) = \frac{\mathbb{r}''(s)}{\kappa(s)} = \left(-\cos\frac{s}{\sqrt{2}}, -\sin\frac{s}{\sqrt{2}}, 0\right)$

$\mathbb{e}_3(s) = \mathbb{e}_1(s) \times \mathbb{e}_2(s) = \left(\frac{1}{\sqrt{2}}\sin\frac{s}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\cos\frac{s}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$.

$-\tau(s)\cdot\mathbb{e}_2(s) = \mathbb{e}_3'(s) = \left(\frac{1}{2}\cos\frac{s}{\sqrt{2}}, \frac{1}{2}\sin\frac{s}{\sqrt{2}}, 0\right)$ より

$\tau(s) = \frac{1}{2}$ である.

6. $\nabla f = \left(\frac{e^x}{1+y^2+z^2}, \frac{-2ye^x}{(1+y^2+z^2)^2}, \frac{-2ze^x}{(1+y^2+z^2)^2}\right)$ より

$\nabla f(0.1.1) = \left(\frac{1}{3}, -\frac{2}{9}, -\frac{2}{9}\right)$ である

7. $\mathrm{div}\,\boldsymbol{a} = \nabla \cdot \boldsymbol{a} = 0 + 0 + 0 = 0$

$\mathrm{rot}\,\boldsymbol{a} = \nabla \times \boldsymbol{a} = (x^2 - 2xz,\ y^2 - 2xy,\ z^2 - 2yz)$ である