応用数学II 解答例

1. $a_0=\frac{1}{\pi}\int_{-\pi}^{\pi}2x-3\,dx=-\frac{3}{\pi}\int_{-\pi}^{\pi}1\,dx=-6$

$a_n=\frac{1}{\pi}\int_{-\pi}^{\pi}(2x-3)\cos nx\,dx=-\frac{3}{\pi}\int_{-\pi}^{\pi}\cos nx\,dx=0$

$b_n=\frac{1}{\pi}\int_{-\pi}^{\pi}(2x-3)\sin nx\,dx=\frac{4}{\pi}\int_{0}^{\pi}x\sin nx\,dx$

$=\frac{4}{\pi}\left[-\frac{1}{n}x\cos nx\right]_0^{\pi}+\frac{4}{\pi n}\int_0^{\pi}\cos nx\,dx$

$=-\frac{4}{n}(-1)^n+\frac{4}{\pi n}\left[\frac{1}{n}\sin nx\right]_0^{\pi}=\frac{4}{n}(-1)^{n+1}$.

$\therefore f(x)\sim-3+\sum_{n=1}^{\infty}\frac{4}{n}(-1)^{n+1}\sin nx$

なお、計算中で偶と奇の計算方法を特に記述せず使っている。

2. $f(x)$は奇より、$a_n=0$.

$b_n=\frac{2}{l}\int_0^{l}\sin\frac{\pi nx}{l}\,dx=\frac{2}{l}\left[\frac{-l}{\pi n}\cos\frac{\pi nx}{l}\right]_0^{l}$

$=-\frac{2}{\pi n}\left((-1)^n-1\right)=\frac{2}{\pi n}\left((-1)^{n+1}+1\right)$.

$\therefore f(x)\sim\sum_{n=1}^{\infty}\frac{2}{\pi n}\left((-1)^{n+1}+1\right)\sin\frac{\pi nx}{l}$

3. $\hat{f}(u) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(t) e^{-iut} dt$

$= \frac{1}{\sqrt{2\pi}} \int_0^1 t e^{-iut} dt$

$= \frac{1}{\sqrt{2\pi}} \left[-\frac{1}{iu} t e^{-iut} \right]_0^1 - \frac{i}{\sqrt{2\pi} \cdot u} \int_0^1 e^{-iut} dt$

$= \frac{1}{\sqrt{2\pi}} \cdot \frac{i}{u} e^{-iu} - \frac{i}{\sqrt{2\pi} \cdot u} \cdot \left[-\frac{1}{iu} e^{-iut} \right]_0^1$

$= \frac{1}{\sqrt{2\pi}} \frac{i}{u} e^{-iu} + \frac{1}{\sqrt{2\pi}} \cdot \frac{1}{u^2} (e^{-iu} - 1)$

4 $\mathbf{a} \times \mathbf{b} = (2\times 3 - 1\times 1, 1\times 2 - 3\times 1, 1\times 1 - 2\times 2)$

$= (5, -1, -3)$.

$\cos\varphi = \frac{(\mathbf{a}\times\mathbf{b})\cdot\mathbf{c}}{|\mathbf{a}\times\mathbf{b}|\cdot|\mathbf{c}|} = \frac{7}{\sqrt{35}\cdot\sqrt{5}} = \frac{\sqrt{7}}{5}$

5, $\mathbf{r}_u = (\cos u \cos v, \cos u \sin v, -\sin u)$

$\mathbf{r}_v = (-\sin u \sin v, \sin u \cos v, 0)$ より

$\mathbf{r}_u \times \mathbf{r}_v = (\sin^2 u \cos v, \sin^2 u \sin v, \cos u \sin u)$

$\therefore |r_u \times r_v| = \sqrt{\sin^4 u \cos^2 v + \sin^4 u \sin^2 v + \cos^2 u \sin^2 u}$

$= \sqrt{\sin^4 u + \cos^2 u \sin^2 u} = |\sin u|.$

$\therefore S = \int_0^{2\pi} \int_0^{\pi} |\sin u| \, du dv$

$= \int_0^{2\pi} \left[-\cos u \right]_0^{\pi} dv = \int_0^{2\pi} 2 \, dv = 4\pi$ //

6. $\mathrm{div}\, \boldsymbol{a} = \nabla \cdot \boldsymbol{a} = \frac{\partial}{\partial x} y^2 z + \frac{\partial}{\partial y} x z^2 + \frac{\partial}{\partial z} x^2 y = 0$

$\mathrm{rot}\, \boldsymbol{a} = \nabla \times \boldsymbol{a} =$

$= \left(\frac{\partial}{\partial y} x^2 y - \frac{\partial}{\partial z} x z^2, \frac{\partial}{\partial z} y^2 z - \frac{\partial}{\partial x} x^2 y, \frac{\partial}{\partial x} x z^2 - \frac{\partial}{\partial y} y^2 z \right)$

$= (x^2 - 2xz, y^2 - 2xy, z^2 - 2yz)$

7. $\nabla f = (2x \sin y \cos z, x^2 \cos y \cos z, -x^2 \sin y \sin z)$ より.

$\nabla f(\pi, 0, 0) = (0, \pi^2, 0)$ である.

また, $\frac{\partial f}{\partial \boldsymbol{e}}(\pi, 0, 0) = (0, \pi^2, 0) \cdot (\alpha, \beta, \gamma) = \beta \pi^2$.

最大となるのは $\boldsymbol{e}$ と ∇f が同じ方向を向くときなので

$\boldsymbol{e} = (0, 1, 0)$ である.