応用数学Ⅱ　解答例

1. $a_n = \dfrac{1}{\pi}\displaystyle\int_{-\pi}^{\pi} f(x)\cos nx\,dx$

$= \dfrac{1}{\pi}\displaystyle\int_{-\pi}^{\pi} (2x-3)\cos nx\,dx$.

$x\cos nx$ は奇 より

$$= \frac{-3}{\pi}\int_{-\pi}^{\pi} \cos nx\,dx = \begin{cases} -6 & n=0 \\ 0 & n \geq 1 \end{cases}.$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} (2x-3)\sin nx\,dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{\pi} 2x\sin nx\,dx = \frac{4}{\pi}\int_{0}^{\pi} x\sin nx\,dx$$

$$= \frac{4}{\pi}\left[-\frac{1}{n}x\cos nx\right]_0^{\pi} + \frac{4}{\pi n}\int_0^{\pi}\cos nx\,dx$$

$$= \frac{4}{n}(-1)^{n+1} + \frac{4}{\pi n}\left[\frac{1}{n}\sin nx\right]_0^{\pi} = \frac{4}{n}(-1)^{n+1}$$

$$\therefore f(x) \sim -3 + \sum_{n=1}^{\infty}\frac{4}{n}(-1)^{n+1}\sin nx$$

2 $\hat{f}(u) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(t) e^{-iut} dt$

$= \frac{1}{\sqrt{2\pi}} \left(\int_{-1}^{0} -t e^{-iut} dt + \int_{0}^{1} t e^{-iut} dt \right)$

$= \frac{1}{\sqrt{2\pi}} \left(\left[\frac{1}{iu} t e^{-iut} \right]_{-1}^{0} + \frac{i}{u} \int_{-1}^{0} e^{-iut} dt \right.$

$\left. + \left[-\frac{1}{iu} t e^{-iut} \right]_{0}^{1} - \frac{i}{u} \int_{0}^{1} e^{-iut} dt \right)$

$= \frac{1}{\sqrt{2\pi}} \left(-\frac{i}{u} e^{iu} + \frac{i}{u} \left[-\frac{1}{iu} e^{-iut} \right]_{-1}^{0} \right.$

$\left. + \frac{i}{u} e^{-iu} - \frac{i}{u} \left[-\frac{1}{iu} e^{-iut} \right]_{0}^{1} \right)$

$= \frac{1}{\sqrt{2\pi}} \left(-\frac{i}{u} e^{iu} + \frac{1}{u^2} e^{iu} - \frac{2}{u^2} + \frac{i}{u} e^{-iu} + \frac{1}{u^2} e^{-iu} \right)$

3. $\mathbf{a} \times \mathbf{b} = \left(\begin{vmatrix} 2 & 1 \\ 1 & 3 \end{vmatrix}, \begin{vmatrix} 1 & 1 \\ 3 & 2 \end{vmatrix}, \begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix} \right) = (5, -1, -3)$

$|\mathbf{a} \times \mathbf{b}| = \sqrt{35} \quad |\mathbf{c}| = \sqrt{5}$

$(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c} = 7 \qquad \therefore \cos\varphi = \frac{7}{\sqrt{35} \cdot \sqrt{5}} = \frac{\sqrt{7}}{5}$

4. $\dot{\mathbb{r}}(t) = (1, 2)$ $|\dot{\mathbb{r}}(t)| = \sqrt{5}$

$\therefore S(t) = \int_0^t \sqrt{5}\, dt = \sqrt{5}\, t$. $t = \frac{S}{\sqrt{5}}$

$\therefore \mathbb{r}(t) = \left(\frac{S}{\sqrt{5}}, 1 + \frac{2}{\sqrt{5}} S \right)$

5. $\mathrm{rot}\, \mathbb{a} = \nabla \times \mathbb{a}$

$= \left(\frac{\partial}{\partial y} x^2 y - \frac{\partial}{\partial z} x z^2, \frac{\partial}{\partial z} y^2 z - \frac{\partial}{\partial x} x^2 y, \frac{\partial}{\partial x} x z^2 - \frac{\partial}{\partial y} y^2 z \right)$

$= (x^2 - 2xz, y^2 - 2xy, z^2 - 2yz)$

$\mathrm{div}\, \mathbb{a} = \nabla \cdot \mathbb{a} = 0 + 0 + 0 = 0$

6. $\mathbb{r}(t) = (x(t), y(t))$ とすると $\dot{\mathbb{r}}(t) = (\dot{x}(t), \dot{y}(t))$

流線の条件より. $\mathbb{a}(\mathbb{r}(t)) = \dot{\mathbb{r}}(t)$ なので

$\begin{cases} \dot{x}(t) = x(t) \\ \dot{y}(t) = 0 \end{cases}$ まず $y(t) = C_2$ となる.

$\dot{x}(t) = x(t)$ をとけば,

$\frac{\dot{x}}{x} = 1$ より $\log x = t + C$

$x = e^{t+C} = C_1 \cdot e^t$ となる

$\therefore \ r(t) = (C_1 e^t, C_2)$