# 応用数学II 解答例

1. $f(x)$ は奇より $a_n = 0$ である

また $b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} x \sin nx \, dx = \frac{2}{\pi}\int_0^{\pi} x \sin nx \, dx$

$$= \frac{2}{\pi}\left[-\frac{1}{n}x\cos nx\right]_0^{\pi} + \frac{2}{\pi n}\int_0^{\pi}\cos nx \, dx$$

$$= -\frac{2}{\pi n}\pi(-1)^n + \frac{2}{\pi n}\left[\frac{1}{n}\sin nx\right]_0^{\pi}$$

$$= \frac{2}{n}(-1)^{n+1}.$$

$\therefore f(x) \sim \sum_{n=1}^{\infty}\frac{2}{n}(-1)^{n+1}\sin nx$ である.

(2) $\sum_{n=1}^{\infty}\left|\frac{2}{n}(-1)^{n+1}\right|^2 = \frac{1}{\pi}\int_{-\pi}^{\pi} x^2 \, dx$

$$= \frac{1}{\pi}\left[\frac{1}{3}x^3\right]_{-\pi}^{\pi} = \frac{2}{3}\pi^2$$

$\therefore 4\sum_{n=1}^{\infty}\frac{1}{n^2} = \frac{2}{3}\pi^2 \qquad \therefore \sum_{n=1}^{\infty}\frac{1}{n^2} = \frac{1}{6}\pi^2$

2. $C(u) = \sqrt{\frac{2}{\pi}}\int_0^{\infty} f(t)\cos ut \, dt = \sqrt{\frac{2}{\pi}}\int_0^{a} 2\cos ut \, dt$

$$= 2\sqrt{\frac{2}{\pi}}\left[\frac{1}{u}\sin ut\right]_0^{a} = 2\sqrt{\frac{2}{\pi}}\frac{1}{u}\sin au$$

3 $\hat{f}(u) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(t) e^{-iut} dt$

$$= \frac{1}{\sqrt{2\pi}} \left( \int_{-1}^{0} -t e^{-iut} dt + \int_{0}^{1} t e^{-iut} dt \right)$$

$$= \frac{1}{\sqrt{2\pi}} \left( \left[ \frac{1}{iu} t e^{-iut} \right]_{-1}^{0} - \frac{1}{iu} \int_{-1}^{0} e^{-iut} dt \right.$$

$$\left. + \left[ -\frac{1}{iu} t e^{-iut} \right]_{0}^{1} + \frac{1}{iu} \int_{0}^{1} e^{-iut} dt \right)$$

$$= \frac{1}{\sqrt{2\pi}} \left( \frac{1}{iu} e^{iu} - \frac{1}{iu} \left[ -\frac{1}{iu} e^{-iut} \right]_{-1}^{0} - \frac{1}{iu} e^{-iu} + \frac{1}{iu} \left[ -\frac{1}{iu} e^{-iut} \right]_{0}^{1} \right)$$

$$= \frac{1}{\sqrt{2\pi}} \left( \frac{i}{u} (e^{iu} + e^{-iu}) - \frac{1}{u^2} (1 - e^{iu}) + \frac{1}{u^2} (e^{-iu} - 1) \right)$$

$$= \frac{1}{\sqrt{2\pi}} \left( \frac{i}{u} (-e^{iu} + e^{-iu}) + \frac{1}{u^2} (e^{iu} + e^{-iu} - 2) \right)$$

4 $\mathbf{a} \times \mathbf{b} = (3-3, 6-1, 1-6) = (0, 5, -5)$

$$|\mathbf{a}\ \mathbf{b}\ \mathbf{c}| = \begin{vmatrix} 1 & 3 & 3 \\ 2 & 1 & 1 \\ 3 & 0 & 3 \end{vmatrix} = 3 + 9 - 9 - 18 = -15$$

5. $\dot{\mathbb{r}}(t) = (-4\sin t,\ 4\cos t,\ 3)$ より

$|\dot{\mathbb{r}}(t)| = \left((-4\sin t)^2 + (4\cos t)^2 + 3^2\right)^{\frac{1}{2}} = 5$ なので

$s(t) = \int_0^t |\dot{\mathbb{r}}(\tau)|\,d\tau = [5\tau]_0^t = 5t \qquad \therefore t = \frac{s}{5}$

$\therefore\ \mathbb{r}(s) = \left(4\cos\frac{s}{5},\ 4\sin\frac{s}{5},\ \frac{3}{5}s\right)$ である。

$\mathbb{e}_1(s) = \mathbb{r}'(s) = \left(-\frac{4}{5}\sin\frac{s}{5},\ \frac{4}{5}\cos\frac{s}{5},\ \frac{3}{5}\right)$

$\mathbb{r}''(s) = \left(-\frac{4}{25}\cos\frac{s}{5},\ -\frac{4}{25}\sin\frac{s}{5},\ 0\right)$ なので

$\kappa(s) = |\mathbb{r}''(s)| = \frac{4}{25}$

$\mathbb{e}_2(s) = \frac{1}{\kappa(s)}\mathbb{r}''(s) = \left(-\cos\frac{s}{5},\ -\sin\frac{s}{5},\ 0\right)$

$\mathbb{e}_3(s) = \mathbb{e}_1(s) \times \mathbb{e}_2(s) =$

$= \left(\frac{3}{5}\sin\frac{s}{5},\ -\frac{3}{5}\cos\frac{s}{5},\ \frac{4}{5}\sin^2\frac{s}{5} + \frac{4}{5}\sin^2\frac{s}{5}\right)$

$= \left(\frac{3}{5}\sin\frac{s}{5},\ -\frac{3}{5}\cos\frac{s}{5},\ \frac{4}{5}\right)$ である

$\mathbb{e}_3'(s) = \left(\frac{3}{25}\cos\frac{s}{5},\ \frac{3}{25}\sin\frac{s}{5},\ 0\right) = -\frac{3}{25}\mathbb{e}_2(s)$ より

$\tau(s) = \frac{3}{25}$ である。

6. $\boldsymbol{r}_u = (\cos u \cos v, \cos u \sin v, -\sin u)$

$\boldsymbol{r}_v = (-\sin u \sin v, \sin u \cos v, 0)$ より

$\boldsymbol{r}_u \times \boldsymbol{r}_v = (\sin^2 u \cos v, \sin^2 u \sin v, \sin u \cdot \cos u)$.

$\therefore |\boldsymbol{r}_u \times \boldsymbol{r}_v| = \sin u$

$\therefore S = \int_0^{\pi} \int_0^{2\pi} \sin u \, dv \, du = 2\pi \left[ -\cos u \right]_0^{\pi} = 4\pi$.

7. $\nabla f = (e^x \sin y, e^x \cos y + z^2 \sin y, -2z \cos y)$ より

$\nabla f(0, \pi, 2) = (0, -1, 4)$ である.