解答例

1 (1) f は偶 より $b_n = 0$. また.

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} x^2 dx = \frac{1}{\pi}\left[\frac{1}{3}x^3\right]_{-\pi}^{\pi} = \frac{2}{3}\pi^2$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} x^2 \cos nx\, dx = \frac{1}{\pi}\left[\frac{1}{n}x^2 \sin nx\right]_{-\pi}^{\pi} - \frac{1}{\pi n}\int_{-\pi}^{\pi} 2x \sin nx\, dx$$

$$= -\frac{2}{\pi n}\left[-\frac{1}{n}x\cos nx\right]_{-\pi}^{\pi} - \frac{2}{\pi n^2}\int_{-\pi}^{\pi}\cos nx\, dx$$

$$= \frac{1}{\pi n^2}\left(4\pi(-1)^n\right) = \frac{4}{n^2}(-1)^n$$ より

$$f(x) \sim \frac{1}{3}\pi^2 + \sum_{n=1}^{\infty}\frac{4}{n^2}(-1)^n \cos nx$$

(2) 上式に $x=0$ を代入すれば

$$0 = \frac{1}{3}\pi^2 + \sum \frac{4}{n^2}(-1)^n$$ より

$$\sum \frac{(-1)^{n+1}}{n^2} = \frac{1}{12}\pi^2$$ となる.

2. $$c_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\, dx = \frac{1}{2\pi}\int_0^{\pi} 2\, dx = 1.$$

$$c_n = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\, e^{-inx} dx = \frac{1}{2\pi}\int_0^{\pi} 2\, e^{-inx} dx$$

$$= \frac{1}{\pi}\left[-\frac{1}{in}e^{-inx}\right]_0^{\pi} = \frac{i}{\pi n}\left((-1)^n - 1\right).$$

$$\therefore f(x) \sim 1 + \sum_{n\neq 0}\frac{i}{\pi n}\left((-1)^n - 1\right)\cdot e^{inx}$$

3 $\hat{f}(u)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}f(x)e^{-iux}dx=\frac{1}{\sqrt{2\pi}}\int_{-4}^{4}(x-4)e^{-iux}dx$

$=\frac{1}{\sqrt{2\pi}}\left(\left[-\frac{1}{iu}(x-4)e^{-iux}\right]_{-4}^{4}-\frac{i}{u}\int_{-4}^{4}e^{-iux}dx\right)$

$=\frac{1}{\sqrt{2\pi}}\left(\frac{8i}{u}e^{4iu}-\frac{i}{u}\left[-\frac{1}{iu}e^{-iux}\right]_{-4}^{4}\right)$

$=\frac{1}{\sqrt{2\pi}}\left(\frac{8i}{u}e^{4iu}+\frac{1}{u^2}(e^{-4iu}-e^{4iu})\right)$ である.

4. $\mathbb{a}\times\mathbb{b}=(5,-1,-3)$, $|\mathbb{a}\times\mathbb{b}|=\sqrt{25+1+9}=\sqrt{35}$, $|\mathbb{c}|=\sqrt{5}$

より $\cos\varphi=\frac{1}{\sqrt{35}}\cdot\frac{1}{\sqrt{5}}(\mathbb{a}\times\mathbb{b})\cdot\mathbb{c}=\frac{1}{5\sqrt{7}}(10-3)=\frac{\sqrt{7}}{5}$.

5 (1) $\dot{\mathbb{r}}(t)=(-3\sin t, 3\cos t, 4)$ より.

$|\dot{\mathbb{r}}(t)|=\sqrt{9\sin^2 t+9\cos^2 t+16}=5$.

$\therefore S(t)=\int_0^t|\dot{\mathbb{r}}(\tau)|d\tau=\int_0^t 5\,d\tau=5t$

$\therefore \mathbb{r}(s)=\left(3\cos\frac{s}{5}, 3\sin\frac{s}{5}, \frac{4}{5}s\right)$ である.

(2) $\mathbb{e}_1(s)=\mathbb{r}'(s)=\left(-\frac{3}{5}\sin\frac{s}{5}, \frac{3}{5}\cos\frac{s}{5}, \frac{4}{5}\right)$.

$\mathbb{r}''(s)=\left(-\frac{3}{25}\cos\frac{s}{5}, -\frac{3}{25}\sin\frac{s}{5}, 0\right)$ より

$|\mathbb{r}''(s)|=\frac{3}{25}$ $\therefore$ $\mathbb{e}_2(s)=\frac{\mathbb{r}''(s)}{|\mathbb{r}''(s)|}=\left(-\cos\frac{s}{5}, -\sin\frac{s}{5}, 0\right)$

$\mathbb{e}_3(s) = \mathbb{e}_1(s) \times \mathbb{e}_2(s) = \left(\frac{4}{5}\sin\frac{s}{5}, -\frac{4}{5}\cos\frac{s}{5}, \frac{3}{5}\right)$ である

6. $\mathbb{r}'(t) = \mathbb{a}(\mathbb{r}(t))$ より　$\mathbb{r}(t) = (x(t), y(t), z(t))$ とすれば

$(x'(t), y'(t), z'(t)) = (1, 1, 1)$ となる.

$x'(t) = 1$ をとけば.　$x(t) = t + C_1$.

$y(t)$, $z(t)$ も同様なので

$$\mathbb{r}(t) = (t + C_1, t + C_2, t + C_3)$$

$$= t(1, 1, 1) + (C_1, C_2, C_3)$$ である.

7. $\nabla f = (2x\sin y\cos z, x^2\cos y\cos z, -x^2\sin y\sin z)$ より

$\nabla f(1, 0, 0) = (0, 1, 0)$ である.

8. $\operatorname{div}\mathbb{a} = \frac{\partial}{\partial x}(y^2 z) + \frac{\partial}{\partial y}(xz^2) + \frac{\partial}{\partial z}(x^2 y) = 0$

$$\operatorname{rot}\mathbb{a} = \left(\frac{\partial}{\partial y}x^2 y - \frac{\partial}{\partial z}xz^2, \frac{\partial}{\partial z}y^2 z - \frac{\partial}{\partial x}x^2 y, \frac{\partial}{\partial x}xz^2 - \frac{\partial}{\partial y}y^2 z\right)$$

$$= (x^2 - 2xz, y^2 - 2xy, z^2 - 2yz)$$ である.