応用数学Ⅱ 解答例

1.(1) $a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} 2x-3\,dx = \frac{1}{\pi}\int_{-\pi}^{\pi} -3\,dx = -\frac{3}{\pi}[x]_{-\pi}^{\pi}$

$= -6$

$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi}(2x-3)\cos nx\,dx = -\frac{3}{\pi}\int_{-\pi}^{\pi}\cos nx\,dx = 0$

$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi}(2x-3)\sin nx\,dx$

$= \frac{2}{\pi}\int_{-\pi}^{\pi} x\sin nx\,dx - \frac{3}{\pi}\int_{-\pi}^{\pi}\sin nx\,dx$

$= \frac{4}{\pi}\int_{0}^{\pi} x\sin nx\,dx$

$= \frac{4}{\pi}\left[-\frac{1}{n}x\cos nx\right]_0^{\pi} + \frac{4}{\pi n}\int_0^{\pi}\cos nx\,dx$

$= -\frac{4}{n}(-1)^n + \frac{4}{\pi n}\left[\frac{1}{n}\sin nx\right]_0^{\pi} = \frac{4}{n}(-1)^{n+1}$

$\therefore f(x) \sim -3 + \sum_{n=1}^{\infty}\frac{4}{n}(-1)^{n+1}\sin nx.$

(2) $\int_{-\pi}^{\pi}(2x-3)^2dx = \int_{-\pi}^{\pi}4x^2-12x+9\,dx$

$= \left[\frac{4}{3}x^3+9x\right]_{-\pi}^{\pi} = \frac{8}{3}\pi^3+18\pi.$

$\therefore \frac{8}{3}\pi^2+18 = \frac{(-6)^2}{2} + \sum_{n=1}^{\infty}\left(\frac{4}{n}(-1)^{n+1}\right)^2.$

$\therefore \sum\frac{16}{n^2} = \frac{8}{3}\pi^2 \qquad \therefore \sum\frac{1}{n^2} = \frac{\pi^2}{6}$

2. $f(x)$ は奇 より $a_n = 0$.

$$b_n = \frac{1}{\ell}\int_{-\ell}^{\ell} f(x)\cdot \sin\frac{\pi n x}{\ell}dx = \frac{2}{\ell}\int_0^{\ell} f(x)\sin\frac{\pi n x}{\ell}dx$$

$$= \frac{2}{\ell}\int_0^{\ell} \sin\frac{\pi n x}{\ell}dx = \frac{2}{\ell}\left[-\frac{\ell}{\pi n}\cos\frac{\pi n x}{\ell}\right]_0^{\ell}$$

$$= -\frac{2}{\pi n}\left((-1)^n - 1\right) = \frac{2}{\pi n}\left(1-(-1)^n\right)$$

$$\therefore\ f(x) \sim \sum_{n=1}^{\infty} \frac{2}{\pi n}\left(1-(-1)^n\right)\sin\frac{\pi n x}{\ell}$$

3. $$\hat{f}(u) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{-itu} f(t)dt = \frac{1}{\sqrt{2\pi}}\int_{-4}^{4} e^{-itu}(t-4)dt$$

$$= \frac{1}{\sqrt{2\pi}}\left[-\frac{1}{iu}e^{-itu}(t-4)\right]_{-4}^{4} + \frac{1}{\sqrt{2\pi}\,iu}\int_{-4}^{4} e^{-itu}dt$$

$$= \frac{1}{\sqrt{2\pi}}\cdot\frac{8i}{u}e^{4iu} + \frac{1}{\sqrt{2\pi}\cdot iu}\left[-\frac{1}{iu}e^{-itu}\right]_{-4}^{4}$$

$$= \frac{1}{\sqrt{2\pi}}\left(\frac{8i}{u}e^{4iu} + \frac{1}{u^2}\left(e^{-4iu} - e^{4iu}\right)\right)$$

4. $$\mathbf{a}\times\mathbf{b} = (8-9,\ 6-4,\ 3-4) = (-1, 2, -1)$$

$$|\mathbf{a}\ \mathbf{b}\ \mathbf{c}| = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 1 & 1 & -1 \end{vmatrix} = \begin{vmatrix} 1 & 2 & 3 \\ 1 & 1 & 1 \\ 1 & 1 & -1 \end{vmatrix} = \begin{vmatrix} 1 & 2 & 3 \\ 1 & 1 & 1 \\ 0 & 0 & -2 \end{vmatrix} = 2$$

5. $\dot{\mathbb{r}}(t) = (1, 2)$ より

$$S = \int_0^t \sqrt{1+2^2}\,dt = \sqrt{5}\,t \qquad \therefore t = \frac{1}{\sqrt{5}}S.$$

$$\therefore \mathbb{r}(S) = \left(\frac{1}{\sqrt{5}}S,\ 1+\frac{2}{\sqrt{5}}S\right)$$

6. $\mathbb{r}_u = (\cos v, \sin v, 1)$

$\mathbb{r}_v = (-u\sin v, u\cos v, 0)$ より

$$\mathbb{r}_u \times \mathbb{r}_v = (-u\cos v, -u\sin v, u)$$

$$\therefore |\mathbb{r}_u \times \mathbb{r}_v| = \sqrt{u^2\cos^2 v + u^2\sin^2 v + u^2} = \sqrt{2}|u| = \sqrt{2}u$$

$$\int_0^1\int_0^{2\pi} \sqrt{2}u\,dv\,du$$

$$= 2\sqrt{2}\pi\int_0^1 u\,du = \sqrt{2}\pi$$

7. $$\nabla f = \left(\frac{e^x}{1+y^2+z^2},\ \frac{-2ye^x}{(1+y^2+z^2)^2},\ \frac{-2ze^x}{(1+y^2+z^2)^2}\right)$$

$$\therefore \nabla f(0.1.1) = \left(\frac{1}{3}, -\frac{2}{9}, -\frac{2}{9}\right)$$

8. $$\mathrm{rot}\,\mathbb{a} = \nabla\cdot\mathbb{a} = \frac{\partial}{\partial x}y^2z + \frac{\partial}{\partial y}xz^2 + \frac{\partial}{\partial z}x^2y = 0$$

$$\mathrm{div}\,\mathbb{a} = \nabla\times\mathbb{a} = (x^2-2xz,\ y^2-2xy,\ z^2-2yz)$$