応用数学II 解答例.

[1] $f(x)$は奇より $a_n=0$ である. また.

$$b_n=\frac{1}{\pi}\int_{-\pi}^{\pi}f(x)\sin nx\,dx=\frac{2}{\pi}\int_0^{\pi}f(x)\sin nx\,dx=\frac{2}{\pi}\int_0^{\pi}\sin nx\,dx$$

$$=\frac{2}{\pi}\cdot\left[\frac{1}{n}\cos nx\right]_0^{\pi}=\frac{2}{\pi n}((-1)^n-1)$$ となる

$\therefore f(x)\sim\sum_{n=1}^{\infty}\frac{2}{\pi n}((-1)^n-1)\sin nx$ である.

[2] $a_0=\frac{1}{\ell}\int_{-\ell}^{\ell}3-2x\ dx=\frac{1}{\ell}\cdot[3x-x^2]_{-\ell}^{\ell}=6$ であり

$$a_n=\frac{1}{\ell}\int_{-\ell}^{\ell}(3-2x)\cos\frac{\pi nx}{\ell}dx=\frac{3}{\ell}\int_{-\ell}^{\ell}\cos\frac{\pi nx}{\ell}dx$$

$$=\frac{3}{\ell}\left[\frac{\ell}{\pi n}\sin\frac{\pi nx}{\ell}\right]_{-\ell}^{\ell}=0$$

$$b_n=\frac{1}{\ell}\int_{-\ell}^{\ell}(3-2x)\sin\frac{\pi nx}{\ell}dx=-\frac{4}{\ell}\int_0^{\ell}x\sin\frac{\pi nx}{\ell}dx$$

$$=-\frac{4}{\ell}\left(\left[\frac{-\ell}{\pi n}x\cos\frac{\pi nx}{\ell}\right]_0^{\ell}+\frac{\ell}{\pi n}\int_0^{\ell}\cos\frac{\pi nx}{\ell}dx\right)$$

$$=\frac{4}{\pi n}\ell\cdot(-1)^n-\frac{4}{\ell}\cdot\frac{\ell}{\pi n}\left[\frac{\ell}{\pi n}\sin\frac{\pi nx}{\ell}\right]_0^{\ell}$$

$$=\frac{(-1)^n4\ell}{\pi n}$$ である.

$\therefore f(x)\sim3+\sum_{n=1}^{\infty}\frac{(-1)^n4\ell}{\pi n}\sin\frac{\pi nx}{\ell}$ である.

[3] $$\hat{f}(u)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}f(t)e^{-iut}dt$$

$$=\frac{1}{\sqrt{2\pi}}\int_0^1 t\,e^{-iut}dt+\frac{1}{\sqrt{2\pi}}\int_{-1}^{0}-t\,e^{-iut}dt$$

$$= \frac{1}{\sqrt{2\pi}}\left(\left[-\frac{1}{iu} t e^{-iut}\right]_0^1 + \frac{1}{iu}\int_0^1 e^{-iut} + \left[\frac{1}{iu} t e^{-iut}\right]_{-1}^0 - \frac{1}{iu}\int_{-1}^0 e^{-iut}\,dt \right)$$

$$= \frac{1}{\sqrt{2\pi}}\left(-\frac{1}{iu}e^{-iu} + \frac{1}{u^2}\left[e^{-iut}\right]_0^1 + \frac{1}{iu}e^{iu} - \frac{1}{u^2}\left[e^{-iut}\right]_{-1}^0\right)$$

$$= \frac{1}{\sqrt{2\pi}}\left(-\frac{1}{iu}e^{-iu} + \frac{1}{iu}e^{iu} + \frac{1}{u^2}e^{-iu} + \frac{1}{u^2}e^{iu} - \frac{2}{u^2}\right)$$ である

$\boxed{4}$ (1) $s(t) = \int_0^t \sqrt{(-3\sin t)^2 + (3\cos t)^2 + 0^2}\,dt = \int_0^t 3\,dt = 3t$ より

$r(s) = \left(3\cos\frac{s}{3}, 3\sin\frac{s}{3}, 2\right)$ である

(2) $r'(s) = \left(-\sin\frac{s}{3}, \cos\frac{s}{3}, 0\right)$

$r''(s) = \left(-\frac{1}{3}\cos\frac{s}{3}, -\frac{1}{3}\sin\frac{s}{3}, 0\right)$ より

$\kappa(s) = \sqrt{\left(-\frac{1}{3}\cos\frac{s}{3}\right)^2 + \left(-\frac{1}{3}\sin\frac{s}{3}\right)^2 + 0^2} = \frac{1}{3}$

(3) $r'''(s) = \left(\frac{1}{9}\sin\frac{s}{3}, -\frac{1}{9}\cos\frac{s}{3}, 0\right)$ より

$\left| r'(s) \ \ r''(s) \ \ r'''(s) \right| = 0$ (第3列が0なので)

$\therefore \tau(s) = 0$.

$\boxed{5}$ $\nabla f = \left(\frac{e^x}{1+y^2+z^2}, \frac{-2ye^x}{(1+y^2+z^2)^2}, \frac{-2ze^x}{(1+y^2+z^2)^2}\right)$ より

$\nabla f(0.1.1) = \left(\frac{1}{3}, -\frac{2}{9}, -\frac{2}{9}\right)$ である.

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また．最大にするeは$\nabla f(0,1,1)$と同じ向きの単位ベクトルなので．

$$|\nabla f(0,1,1)| = \sqrt{\left(\frac{1}{3}\right)^2 + \left(-\frac{2}{9}\right)^2 + \left(-\frac{2}{9}\right)^2} = \frac{1}{9}\sqrt{17}.$$

$$\therefore e = \frac{9}{\sqrt{17}}\left(\frac{1}{3}, -\frac{2}{9}, -\frac{2}{9}\right) = \frac{1}{\sqrt{17}}(3, -2, -2)$$ である．

6 $r(t) = (x(t), y(t), z(t))$ とおくと．流線は

$a(r(t)) = \dot{r}(t)$ をみたす．これより

$(x(t), y(t), z(t)) = (\dot{x}(t), \dot{y}(t), \dot{z}(t))$ となる．ここで

$x(t) = \dot{x}(t)$ を解けば，変数分離形なので．

$\frac{1}{x}dx = dt$ より $\log x = t + c_1$，$x = c_1 e^t$ (e^{c_1}をc₁でおきかえた)となる

同様に $y = c_2 e^t$，$z = c_3 e^t$ となり

$r(t) = e^t(c_1, c_2, c_3)$ となる．

7 $\nabla \cdot a = 0 + 0 + 0 = 0$.

$\nabla \times a = (x^2 - 2xz, y^2 - 2xy, z^2 - 2yz)$ である．