解答

1 $6x$ が奇, -4 が偶 であることを考えれば

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} 6x-4\,dx = \frac{2}{\pi}\int_0^{\pi} -4\,dx = -8$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} (6x-4)\cos nx\,dx = \frac{2}{\pi}\int_0^{\pi} -4\cos nx\,dx$$

$$= -\frac{8}{\pi}\left[\frac{1}{n}\sin nx\right]_0^{\pi} = 0 \qquad (n \neq 0)$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} (6x-4)\sin nx\,dx = \frac{12}{\pi}\int_0^{\pi} x\sin nx\,dx$$

$$= \frac{12}{\pi}\left[-\frac{x}{n}\cos nx\right]_0^{\pi} + \frac{12}{\pi n}\int_0^{\pi}\cos nx\,dx$$

$$= -\frac{12}{n}\cdot(-1)^n + \frac{12}{\pi n}\left[\frac{1}{n}\sin nx\right]_0^{\pi} = \frac{12}{n}(-1)^{n+1}$$ となる.

$$\therefore f(x) \sim -4 + \sum_{n=1}^{\infty}\frac{12}{n}(-1)^{n+1}\sin nx$$ である.

2 $$c_0 = \frac{1}{2\pi}\int_0^{\pi} 2\,dx = 1.$$

$$c_n = \frac{1}{2\pi}\int_0^{\pi} 2\cdot e^{-inx}\,dx = \frac{1}{\pi}\left[\frac{1}{-in}e^{-inx}\right]_0^{\pi}$$

$$= \frac{i}{\pi n}\left((-1)^n - 1\right)$$ となる. これより

$$f(x) \sim 1 + \sum_{n\neq 0}\frac{i}{\pi n}\left((-1)^n-1\right)e^{inx}$$ である.

3 $\hat{f}(u) = \frac{1}{\sqrt{2\pi}} \int_{-4}^{4} (t-4) \cdot e^{-iut} dt$

$= \frac{1}{\sqrt{2\pi}} \left[-\frac{1}{iu}(t-4)e^{-iut} \right]_{-4}^{4} + \frac{1}{\sqrt{2\pi}} \cdot \frac{1}{iu} \cdot \int_{-4}^{4} e^{-iut} dt$

$= \frac{1}{\sqrt{2\pi}} \cdot \frac{i}{u}(+8) \cdot e^{4iu} + \frac{1}{\sqrt{2\pi}} \cdot \frac{-i}{u} \left[-\frac{1}{iu} e^{-iut} \right]_{-4}^{4}$

$= \frac{1}{\sqrt{2\pi}} \left(\frac{8i}{u} e^{4iu} + \frac{1}{u^2}\left(e^{-4iu} - e^{4iu}\right) \right)$ となる

4 $a \times b = \left(\begin{vmatrix} 1 & -1 \\ 5 & 1 \end{vmatrix}, \begin{vmatrix} -1 & 2 \\ 1 & 3 \end{vmatrix}, \begin{vmatrix} 2 & 1 \\ 3 & 5 \end{vmatrix} \right)$

$= (6, -5, 7)$

$|a\ b\ c| = \begin{vmatrix} 2 & 1 & -1 \\ 3 & 5 & 1 \\ -1 & -2 & 3 \end{vmatrix} = 30 + 6 - 1 - 5 - 9 + 4 = 25$ である.

5 $\dot{r}(t) = (-a\sin t, a\cos t, b)$ より

$|\dot{r}(t)| = \sqrt{(-a\sin t)^2 + (a\cos t)^2 + b^2} = \sqrt{a^2+b^2}$ となる. これより

$s(t) = \int_0^t |\dot{r}(\tau)| d\tau = \int_0^t \sqrt{a^2+b^2} d\tau = \sqrt{a^2+b^2} \cdot t$ である.

$\therefore r(s) = \left(a\cos\frac{s}{\sqrt{a^2+b^2}}, a\sin\frac{s}{\sqrt{a^2+b^2}}, \frac{bs}{\sqrt{a^2+b^2}} \right)$ である.

また. $r'(s) = \frac{1}{\sqrt{a^2+b^2}} \left(-a\sin\frac{s}{\sqrt{a^2+b^2}}, a\cos\frac{s}{\sqrt{a^2+b^2}}, b \right)$

$r''(s) = \frac{1}{a^2+b^2} \left(-a\cos\frac{s}{\sqrt{a^2+b^2}}, -a\sin\frac{s}{\sqrt{a^2+b^2}}, 0 \right)$ より

$\kappa(s) = |r''(s)| = \frac{1}{a^2+b^2} \cdot \sqrt{\left(-a\cos\frac{s}{\sqrt{a^2+b^2}}\right)^2 + \left(-a\sin\frac{s}{\sqrt{a^2+b^2}}\right)^2} = \frac{a}{a^2+b^2}$ である.

[6] $\nabla f = (2x\sin y\cos z,\ x^2\cos y\cos z,\ x^2\sin y\cdot(-\sin z))$ より

$\nabla f(\pi, 0, 0) = (0, \pi^2, 0)$ である.

$\frac{\partial f}{\partial e}(\pi, 0, 0)$ を最大にする e は, $\nabla f(\pi, 0, 0)$ と同じ方向の単位ベクトルなので,

$\frac{\nabla f(\pi, 0, 0)}{|\nabla f(\pi, 0, 0)|} = \frac{1}{\pi^2}(0, \pi^2, 0) = (0, 1, 0)$ である.

[7] $\mathrm{div}\cdot f = \nabla\cdot f = \frac{2xy}{(x^2+y^2)^2} - \frac{2xy}{(x^2+y^2)^2} + 2\cos 2z\cdot e^{\sin 2z} = 2\cos 2z\cdot e^{\sin 2z}$

であり, また

$\mathrm{rot} f = \nabla\times f$

$= \left(\frac{\partial}{\partial y}e^{\sin 2z} - \frac{\partial}{\partial z}\cdot\frac{x}{x^2+y^2},\ \frac{\partial}{\partial z}\left(-\frac{y}{x^2+y^2}\right) - \frac{\partial}{\partial x}e^{\sin 2z},\ \frac{\partial}{\partial x}\left(\frac{x}{x^2+y^2}\right) - \frac{\partial}{\partial y}\left(-\frac{y}{x^2+y^2}\right)\right)$

$= \left(0, 0, \frac{1}{x^2+y^2} - \frac{2x^2}{(x^2+y^2)^2} + \frac{1}{x^2+y^2} - \frac{2y^2}{(x^2+y^2)^2}\right) = (0, 0, 0)$

となる.