

微分・積分公式

	微分	積分
x^a		
$\sin x$		
$\cos x$		
$\tan x$		*
$\log x$		*
e^x		
$f(g(x))$		*
$f(x)g(x)$		*
$\frac{f(x)}{g(x)}$		*
$\int f(x)g'(x)dx$	*	
$\int f(g(x))g'(x)dx$	*	
$\int \frac{f'(x)}{f(x)}dx$	*	
$\int f'(x)e^{f(x)}dx$	*	

微分・積分公式

	微分	積分
x^a	$a \cdot x^{a-1}$	$\frac{1}{a+1} x^{a+1} \quad (a \neq -1)$ $\log x \quad (a = -1)$
$\sin x$	$\cos x$	$-\cos x$
$\cos x$	$-\sin x$	$\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$	*
$\log x$	$\frac{1}{x}$	*
e^x	e^x	e^x
$f(g(x))$	$g'(x) \cdot f'(g(x))$	*
$f(x)g(x)$	$f(x) \cdot g'(x) + f'(x) \cdot g(x)$	*
$\frac{f(x)}{g(x)}$	$\frac{f(x) \cdot g'(x) - f'(x) \cdot g(x)}{g(x)^2}$	*
$\int f(x)g'(x)dx$	*	$f(x) \cdot g(x) - \int f'(x) \cdot g(x) \cdot dx$
$\int f(g(x))g'(x)dx$	*	$\int f(t) \cdot dt \quad (t = g(x))$
$\int \frac{f'(x)}{f(x)} dx$	*	$\log f(x)$
$\int f'(x)e^{f(x)} dx$	*	$e^{f(x)}$

演習問題

Ⅰ 次の式の微分と不定積分を求めよ。

(1) $(x+1)^4$

(2) $\sin(2x+3)$

(3) e^{-x+1}

(4) $x \cdot \cos x$

Ⅱ 次の式の微分を求めよ

(1) $\frac{x^2-1}{x^2+1}$

(2) 2^x

(3) $\cos(x^2+1)$

(4) $\log(\log x)$

Ⅲ 次の式的不定積分を求めよ

(1) $x \cdot \sqrt{1-x^2}$

(2) $x \cdot e^x$

(3) $\frac{e^x}{e^x+1}$

(4) $\cos x \cdot e^{\sin x}$

解答

$$\square (1) \{(x+1)^4\}' = 4 \cdot (x+1)^3$$

$$\int (x+1)^4 dx = \frac{1}{5} (x+1)^5 \quad (\text{積分定数 } C \text{ は省略})$$

$$(2) \{\sin(2x+3)\}' = 2 \cdot \cos(2x+3)$$

$$\int \sin(2x+3) dx = -\frac{1}{2} \cos(2x+3)$$

$$(3) \{e^{-x+1}\}' = -e^{-x+1}$$

$$\int e^{-x+1} dx = -e^{-x+1}$$

$$(4) \{x \cdot \cos x\}' = \cos x - x \cdot \sin x$$

$$\int x \cdot \cos x dx = x \cdot \sin x - \int \sin x \cdot dx$$

$$= x \cdot \sin x + \cos x$$

$$\square (1) \left\{ \frac{x^2-1}{x^2+1} \right\}' = \frac{2x(x^2+1) - 2x(x^2-1)}{(x^2+1)^2} = \frac{4x}{(x^2+1)^2}$$

$$(2) \{2^x\}' = \{e^{\log 2 \cdot x}\}' = \log 2 \cdot e^{\log 2 \cdot x} = \log 2 \cdot 2^x$$

$$(3) \{\cos(x^2+1)\}' = -2x \cdot \sin(x^2+1)$$

$$(4) \{\log(\log x)\}' = \frac{1}{x} \cdot \frac{1}{\log x}$$

$$\square (1) \int x \cdot \sqrt{1-x^2} dx = -\frac{1}{2} \int -2x \sqrt{1-x^2} dx$$

$t = 1-x^2$ とする

$$= -\frac{1}{2} \int \sqrt{t} dt = -\frac{1}{3} t^{\frac{3}{2}} = -\frac{1}{3} (1-x^2)^{\frac{3}{2}}$$

$$(2) \int x \cdot e^x dx = x \cdot e^x - \int e^x dx = x \cdot e^x - e^x$$

$$(3) \int \frac{e^x}{e^x+1} dx = \log(e^x+1)$$

$$(4) \int \cos x \cdot e^{\sin x} dx = e^{\sin x}$$