期末試験解答

1. $f(x)$ は奇関数なので $a_n=0$

$$b_n=\frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx = \frac{2}{\pi}\cdot\int_0^{\pi}\sin nx\,dx = \frac{2}{\pi}\left[-\frac{1}{n}\cos nx\right]_0^{\pi}$$

$$= -\frac{2}{n\pi}(\cos n\pi - 1) = \frac{2}{n\pi}(1-(-1)^n)$$

$\therefore f(x) \sim \sum_{n=1}^{\infty}\frac{2}{n\pi}(1-(-1)^n)\cdot\sin nx$ である.

2. $$F(\tau) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x)\cdot e^{-i\tau x}\,dx$$

$$= \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{0} e^{ax}\cdot e^{-i\tau x}\,dx + \frac{1}{\sqrt{2\pi}}\int_0^{\infty} e^{-ax}e^{-i\tau x}\,dx$$

$$= \frac{1}{\sqrt{2\pi}}\left[\frac{1}{a-i\tau}e^{ax-i\tau x}\right]_{-\infty}^{0} + \frac{1}{\sqrt{2\pi}}\left[\frac{1}{-a-i\tau}e^{-ax-i\tau x}\right]_0^{\infty}$$

$$= \frac{1}{\sqrt{2\pi}}\left(\frac{1}{a-i\tau}+\frac{1}{a+i\tau}\right) = \frac{\sqrt{2}\,a}{\sqrt{\pi}(a^2+\tau^2)}$$ である.

3. $$\sqrt{2\pi}H(\tau) = \int_{-\infty}^{\infty} h(x)e^{-i\tau x}dx = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(t)\cdot g(x-t)\cdot e^{-i\tau x}\,dt\,dx$$

$$= \int_{-\infty}^{\infty} f(t)\cdot\int_{-\infty}^{\infty} g(x-t)e^{-i\tau x}\cdot dx\cdot dt$$

ここで $x-t=y$ とおけば. $dx=dy$ より

$$= \int_{-\infty}^{\infty} f(t)\cdot\int_{-\infty}^{\infty} g(y)\cdot e^{-i\tau y}\cdot e^{-i\tau t}\cdot dy\,dt$$

$$= \int_{-\infty}^{\infty} f(t)\cdot\sqrt{2\pi}\cdot G(\tau)\cdot e^{-i\tau t}dt$$

$$= 2\pi\cdot F(\tau)\cdot G(\tau)$$ より 証明された.

4. $S(\tau)=\sqrt{\frac{2}{\pi}}\int_0^{\infty} f(x)\sin\tau x\,dx$

$=\sqrt{\frac{2}{\pi}}\int_0^{a}\left(1-\frac{x}{a}\right)\sin\tau x\,dx$

$=\sqrt{\frac{2}{\pi}}\left[-\frac{1}{\tau}\left(1-\frac{x}{a}\right)\cdot\cos\tau x\right]_0^a+\sqrt{\frac{2}{\pi}}\int_0^a-\frac{1}{a\tau}\cdot\cos\tau x\,dx$

$=\sqrt{\frac{2}{\pi}}\frac{1}{\tau}-\sqrt{\frac{2}{\pi}}\cdot\frac{1}{a\tau}\cdot\left[\frac{1}{\tau}\sin\tau x\right]_0^a$

$=\sqrt{\frac{2}{\pi}}\left(\frac{1}{\tau}-\frac{1}{a\tau^2}\sin a\tau\right)$ である

5. $a\times b=\left(\begin{vmatrix}2&-1\\-6&2\end{vmatrix},\begin{vmatrix}-1&4\\2&-9\end{vmatrix},\begin{vmatrix}4&2\\-9&-6\end{vmatrix}\right)=(-2,1,-6)$

6. $\dot{r}(t)=(-\cos t,\sin t,1)$ より

$s(t)=\int_0^t|\dot{r}(t)|\,dt=\int_0^t\sqrt{2}\,dt=\sqrt{2}t$ である $\therefore\ t=\frac{1}{\sqrt{2}}s$.

$r(s)=\left(1-\sin\frac{s}{\sqrt{2}},1-\cos\frac{s}{\sqrt{2}},\frac{s}{\sqrt{2}}\right)$ より

$\mathbb{t}(s)=r'(s)=\frac{1}{\sqrt{2}}\left(-\cos\frac{s}{\sqrt{2}},\sin\frac{s}{\sqrt{2}},1\right)$.

$r''(s)=\frac{1}{2}\left(\sin\frac{s}{\sqrt{2}},\cos\frac{s}{\sqrt{2}},0\right)$ より $\kappa(s)=|r''(s)|=\frac{1}{2}$

$r'''(s)=\frac{1}{2\sqrt{2}}\left(\cos\frac{s}{\sqrt{2}},-\sin\frac{s}{\sqrt{2}},0\right)$ より

$\tau(s)=\frac{1}{|\kappa(s)|^2}\cdot\frac{1}{\sqrt{2}}\cdot\frac{1}{2}\cdot\frac{1}{2\sqrt{2}}\cdot\begin{vmatrix}-\cos\frac{s}{\sqrt{2}}&\sin\frac{s}{\sqrt{2}}&1\\\sin\frac{s}{\sqrt{2}}&\cos\frac{s}{\sqrt{2}}&0\\\cos\frac{s}{\sqrt{2}}&-\sin\frac{s}{\sqrt{2}}&0\end{vmatrix}=-\frac{1}{2}$ である.

7. $\nabla f=(2x,2y,2z)$ より $(2,4,6)$ が等位面と垂直なベクトルになる

$\therefore\ n=\frac{1}{\sqrt{14}}(1,2,3)$ である.