No.

応用数学II　期末試験　解答

1.　$f(x)$ は奇関数 より　$a_n = 0$.

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cdot \sin nx \, dx = \frac{2}{\pi}\int_0^{\pi} f(x)\cdot \sin nx \, dx$$

$$= \frac{1}{\pi}\int_0^{\pi} (\pi - x)\cdot \sin nx \, dx$$

$$= \frac{1}{\pi}\left(\left[-\frac{1}{n}(\pi - x)\cdot \cos nx\right]_0^{\pi} + \int_0^{\pi} - \cos nx \cdot dx\right.$$

$$= \frac{1}{n} - \left[\frac{1}{n}\sin nx\right]_0^{\pi} = \frac{1}{n}.$$

$$\therefore f(x) = \sum_{n=1}^{\infty} \frac{1}{n}\cdot \sin nx$$ である.

2.　$$C_n = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\cdot e^{-inx}\, dx = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{(2-in)x+3}\, dx$$

$$= \frac{1}{2\pi}\left[\frac{1}{2-in}\cdot e^{(2-in)x}\cdot e^3\right]_{-\pi}^{\pi}$$

$$= \frac{1}{2\pi}\cdot e^3\cdot \frac{1}{2-in}\cdot\left(e^{2\pi - in\pi} - e^{-2\pi + in\pi}\right)$$

$$= \frac{1}{2\pi} e^3\cdot \frac{1}{2-in}\cdot(-1)^n\cdot\left(e^{2\pi} - e^{-2\pi}\right)$$

$$\therefore f(x) = \frac{e^3}{2\pi}\cdot \sum_{n=-\infty}^{\infty} \frac{(-1)^n}{2-in}\left(e^{2\pi} - e^{-2\pi}\right)\cdot e^{inx}$$ である.

3.　$$F(\tau) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x)\cdot e^{-i\tau\xi}\cdot d\xi.$$

$$= \frac{1}{\sqrt{2\pi}}\left(\int_{-a}^{0} - e^{-i\tau\xi} d\xi + \int_0^{a} e^{-i\tau\xi} d\xi\right)$$

$$= \sqrt{\frac{1}{2\pi}}\left(\left[-\frac{1}{-i\tau} e^{-i\tau\xi}\right]_{-a}^{0} + \left[\frac{1}{-i\tau} e^{-i\tau\xi}\right]_0^{a}\right)$$

$$= \frac{1}{\sqrt{2\pi}}\cdot\frac{1}{i\tau}\cdot\left(1 - e^{i\tau a} - e^{-i\tau a} + 1\right)$$

$$= \frac{i}{\sqrt{2\pi}\cdot\tau}\left(e^{i\tau a} + e^{-i\tau a} - 2\right) \qquad \left(= \frac{i\sqrt{2}}{\sqrt{\pi}\cdot\tau}\left(\cos a\tau - 1\right)\right)$$

である

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4. $b(t) = (b_1(t), b_2(t), b_3(t))$ とおくと. $a(t) = k \cdot b(t)$ より

$$a(t) = (k \cdot b_1(t), k \cdot b_2(t), k \cdot b_3(t))$$ とできる.

これより. $a(t) \times b(t)$

$$= \left(\begin{vmatrix} kb_2(t) & kb_3(t) \\ b_2(t) & b_3(t) \end{vmatrix}, \begin{vmatrix} kb_3(t) & kb_1(t) \\ b_3(t) & b_1(t) \end{vmatrix}, \begin{vmatrix} kb_1(t) & kb_2(t) \\ b_1(t) & b_2(t) \end{vmatrix} \right)$$

$= (0, 0, 0) = 0$ となり証明された.

5. まず. $\dot{r}(t) = (\cos t, -\sin t, 1)$ である.

$t=0$ から $t=t$ までの曲線の長さ $s(t)$ は.

$$s(t) = \int_0^t |\dot{r}(t)|\, dt = \int_0^t \sqrt{2}\, dt = \sqrt{2}t$$ となる.

$\therefore\ t = \frac{1}{\sqrt{2}} s$ とおけば, 曲線は $r(s) = \left(\sin\frac{s}{\sqrt{2}}, \cos\frac{s}{\sqrt{2}}, \frac{s}{\sqrt{2}}\right)$ となる

$\therefore\ \mathbb{t} = r'(s) = \left(\frac{1}{\sqrt{2}}\cos\frac{s}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\sin\frac{s}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ である.

さらに $r''(s) = \left(-\frac{1}{2}\sin\frac{s}{\sqrt{2}}, -\frac{1}{2}\cos\frac{s}{\sqrt{2}}, 0\right)$ より.

$$\kappa = |r''(s)| = \frac{1}{2}$$

$$\mathbb{n} = \frac{1}{\kappa} \cdot r''(s) = \left(-\sin\frac{s}{\sqrt{2}}, -\cos\frac{s}{\sqrt{2}}, 0\right)$$ である.

また. $\mathbb{b} = \mathbb{t} \times \mathbb{n} = \left(\frac{1}{\sqrt{2}}\cos\frac{s}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\sin\frac{s}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right) \times \left(-\sin\frac{s}{\sqrt{2}}, -\cos\frac{s}{\sqrt{2}}, 0\right)$

$= \left(\frac{1}{\sqrt{2}}\cos\frac{s}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\sin\frac{s}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$ である.

$$r'''(s) = \left(-\frac{1}{2\sqrt{2}}\cos\frac{s}{\sqrt{2}}, \frac{1}{2\sqrt{2}}\sin\frac{s}{\sqrt{2}}, 0\right)$$ より

$$\tau = \frac{1}{\kappa^2} \begin{vmatrix} \frac{1}{\sqrt{2}}\cos\frac{s}{\sqrt{2}} & -\frac{1}{\sqrt{2}}\sin\frac{s}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{2}\sin\frac{s}{\sqrt{2}} & -\frac{1}{2}\cos\frac{s}{\sqrt{2}} & 0 \\ -\frac{1}{2\sqrt{2}}\cos\frac{s}{\sqrt{2}} & \frac{1}{2\sqrt{2}}\sin\frac{s}{\sqrt{2}} & 0 \end{vmatrix} = -\frac{1}{2}$$ となる.