

# Talk Titles and Abstracts

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**Title:** Norm conditions for separability in  $M_m \otimes M_n$

**Invited Speaker:** Tsuyoshi Ando (ando@es.hokudai.ac.jp), Hokkaido University, Sapporo, Japan

**Abstract:** An element  $\mathbf{S}$  of the tensor product  $M_m \otimes M_n$  is said to be separable if it admits a (separable) decomposition

$$\mathbf{S} = \sum_p X_p \otimes Y_p \quad \exists 0 \leq X_p \in M_m, \exists 0 \leq Y_p \in M_n.$$

This decomposition is not unique. We present some conditions on suitable norms of  $\mathbf{S}$  which guarantee its separability.

Even when separability of  $\mathbf{S}$  is guaranteed by some method, its separable decomposition itself is difficult to construct. We present a general condition which makes it possible to find a way of an effective separable decomposition.

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**Title:** Quantum  $f$ -divergences in von Neumann algebras

**Invited Speaker:** Fumio Hiai (hiai.fumio@gmail.com), Tohoku University, Sendai, Japan.

**Abstract:** This talk is a comprehensive survey on recent developments of quantum divergences in general von Neumann algebras, including standard  $f$ -divergences, maximal  $f$ -divergences, and Rényi type divergences, whose mathematical backgrounds are the Tomita-Takesaki theory, Araki's relative modular operator, and Haagerup's  $L^p$ -spaces. Standard  $f$ -divergences were formerly studied by Petz in a bit more general formula with name quasi-entropy, whose most familiar one is the relative entropy initiated by Umegaki and extended to general von Neumann algebras by Araki.

I extend Kosaki's variational expression of the relative entropy to an arbitrary standard  $f$ -divergence, from which most important properties of standard  $f$ -divergences follow immediately. Then I discuss the Rényi divergence and the sandwiched Rényi divergence in von Neumann algebras, which have recently been developed by Jenčová and Berta-Scholz-Tomamichel. Finally, I treat maximal  $f$ -divergences and discuss their definition, integral expression, and comparison with standard  $f$ -divergences.

This talk is dedicated to the memory of Dénes Petz.

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**Title:** Bottleneck matrices and combinatorial Perron parameters of trees

**Speaker:** Enide Andrade (enide@ua.pt), University of Aveiro, Aveiro, Portugal.

**Co-authors:** Lorenzo Ciardo and Geir Dahl, University of Oslo, Norway.

**Abstract:** We present the notion of combinatorial Perron value (CPV) and a new parameter which gives a new lower bound on the spectral radius of the bottleneck matrix of a rooted tree. Some properties of these two parameters are shown. These questions were motivated by the concept of algebraic connectivity and the *spectral center* of a tree (the so called characteristic set). A certain extension property for the CPV is shown and it allows us to define a new center concept for caterpillars.

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**Title:** Symmetric, Hankel-symmetric, and Centrosymmetric Doubly Stochastic Matrices

**Speaker:** Lei Cao (leicaomath@gmail.com), Georgian Court University, Lakewood, USA.

**Co-author:** Richard A. Braulti, University of Wisconsin - Madison, USA

**Abstract:** We investigate convex polytopes of doubly stochastic matrices having special structures: symmetric, Hankel symmetric, centrosymmetric, and both symmetric and Hankel symmetric. We determine dimensions of these polytopes and classify their extreme points. We also determine a basis of the real vector spaces generated by permutation matrices with these special structures.

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**Title:** A note on Positivstellensätze for matrix polynomials

**Speaker:** Hoa-Binh T. Du (hoabinhdsp@gmail.com), Quy Nhon University, Binh Dinh, Viet Nam.

**Co-author:** Le Cong Trinh (lecongtrinh@qnu.edu.vn), Quy Nhon University, Binh Dinh, Viet Nam.

**Abstract:** In this talk, we give a note on the relation of the positivity of polynomial matrices and their homogenizations on basic closed semi-algebraic sets. Base on this relation, we extend Putinar-Vasilescu's Positivstellensatz, in particular, Reznick's Positivstellensatz, and Dickinson-Povh's Positivstellensatz to (not necessarily homogeneous) polynomial matrices.

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**Title:** A short proof of an inverse eigenvalue problem for Jacobi matrices

**Speaker:** Carlos M. da Fonseca (ando@es.hokudai.ac.jp), Kuwait University

**Abstract:** An alternating sign matrix  $A$  is a square  $(0, \pm 1)$ -matrix such that, ignoring the 0's, in each row and column, the +1's and -1's alternate, beginning and ending with a +1. Any matrix obtained from  $A$  by replacing some 0's by +1 is a completion of  $A$ . In this talk the author shows that any bordered-permutation  $(0, -1)$ -matrix can be completed to an alternating sign matrix. The uniqueness case is fully characterize.

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**Title:** Multivariate convex operator means

**Speaker:** Frank Hansen (frank.hansen@m.tohoku.ac.jp), Tohoku University, Sendai, Japan.

**Abstract:** The dominant method for introducing multivariate operator means is to express them as fixpoints under a contraction with respect to the Thompson metric. Although this method is powerful, it crucially depends on monotonicity. We are developing a technique to prove the existence of multivariate operator means that are not necessarily monotone. This gives rise to an entire new class of convex non-monotonic multivariate operator means.

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**Title:** Nonsingular subspaces of  $M_n(\mathbb{F})$ ,  $\mathbb{F}$  a field.

**Speaker:** Schindranath Jayaraman (sachindranathj@iisertvm.ac.in), Indian Institute of Science Education and Research - Thiruvananthapuram, Vithura, Thiruvananthapuram, Kerala, India.

**Co-author:** Dr. Himadri Mukherjee (himadrim@goa.bits-pilani.ac.in), Birla Institute of Technology and Science, Pilani - Goa, Sancoale, Goa, India.

**Abstract:** For a field  $\mathbb{F}$ , a subspace  $\mathcal{V}$  of  $M_n(\mathbb{F})$  is said to be nonsingular if every nonzero element of  $\mathcal{V}$  is nonsingular. When  $\mathbb{F} = \mathbb{C}$ , any such subspace has dimension at most 1 and when  $\mathbb{F} = \mathbb{R}$ , a nonsingular subspace of dimension  $n$  in  $M_n(\mathbb{R})$  will exist if and only if  $n = 2, 4, 8$ . Our objective is to understand the structure of nonsingular subspaces of dimension  $n$  in  $M_n(\mathbb{R})$ . Connections with a specific linear preserver problem will be pointed out.

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## References

- [1] B. Corbas and G. D. Williams. *Congruence of two-dimensional subspaces in  $M_2(\mathbb{k})$  (characteristic  $\neq 2$ )*. Pacific Journal of Mathematics, 188(2):225–235, 1999.
- [2] J. Dorsey, T. Gannon, N. Jacobson, C. R. Johnson and M. Turnansky. *Linear preservers of semi-positive matrices*. Linear and Multilinear Algebra, (2015), DOI: 10.1080/03081087.2015.1122723.
- [3] L. Rodman and P. Semrl. *A localization technique for linear preserver problems*. Linear Algebra and its Applications, 433:2257–2268, 2010.
- [4] C. de Seguins-Pazzis. *The singular linear preservers of nonsingular matrices*. Linear Algebra and its Applications, 433(2):483–490, 2010.

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**Title:** A unified approach for the Schrodinger equations in quaternionic and split quaternionic mechanics

**Speaker:** Jiang Tongsong (jiangtongsong@sina.com), Heze University, Heze Shandong 274015, China.

**Abstract:** This paper aims to present, in a unified manner, algebraic techniques for Schrodinger equations which are valid on both the algebras and split quaternions. This paper studies generalized quaternion Schrodinger equation by means of a complex representation of generalized quaternion matrices, and give algebraic techniques for the generalized quaternionic Schrodinger equation. This paper derives a unification of algebraic techniques for Schrodinger equations in quaternionic and split quaternionic mechanics. This paper also give algebraic techniques for generalized quaternion matrices, and derives a unification of algebraic techniques for eigenvalues on quaternion and split quaternion matrices.

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**Title:** Separability criteria of multi-qubit states in terms of diagonal and anti-diagonal entries

**Speaker:** Seung-Hyeok Kye (kye@snu.ac.kr), Seoul National University, Seoul, Korea

**Co-authors:** Kil-Chan Ha (Sejong University) and Kyung Hoon Han (The University of Suwon)

**Abstract:** We give separability criteria for general multi-qubit states in terms of diagonal and anti-diagonal entries. We define two numbers which are obtained from diagonal and anti-diagonal entries, respectively, and compare them to get criteria. They give rise to characterizations of separability when all the entries are zero except for diagonal and anti-diagonal, like Greenberger-Horne-Zeilinger diagonal states. The criteria is strong enough to get nonzero volume of entanglement with positive partial transposes.

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**Title:** Karcher equations and geometric mean tensor

**Speaker:** Hosoo Lee (hosoolee@skku.edu), Sungkyunkwan University, Korea.

**Co-authors:** Sejong Kim (Chungbuk National University) and Yongdo Lim (Sungkyunkwan University).

**Abstract:** One of the important property of the Cartan mean on the Riemannian manifold of positive definite matrices is the repetition invariancy from its defining equation

$$\Lambda_n(A_1, \dots, A_n) = \Lambda_{nk}(A_1, \dots, A_n, \dots, A_1, \dots, A_n),$$

where the number of blocks is  $k$ . This is a necessary condition for the extension of a mean to a barycentric map of Borel probability measures.

In this talk, we introduce a new construction scheme of repetition invariant geometric means from a geometric mean tensor and obtain corresponding contractive barycentric maps of integrable Borel probability measures. They retain most of the properties of the Cartan barycenter. Inequalities and formula from the derived geometric means including the Yamazaki inequality, Lie-Trotter formula and unitarily invariant norm inequalities are presented.

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**Title:** Some log-majorizations and a determinantal inequality

**Speaker:** Rute Lemos (rute@ua.pt), CIDMA, University of Aveiro, Aveiro, Portugal.

**Co-author:** Graça Soares, CMAT-UTAD, University of Trás-os-Montes e Alto Douro, Vila Real, Portugal.

**Abstract:** An eigenvalue inequality involving a matrix connection and its dual is established, and some log-majorization type results are obtained. In particular, some eigenvalues inequalities considered by F. Hiai and M. Lin and a singular values inequality by L. Zou are revisited. A reformulation of the inequality  $\det(A + U^*B) \leq \det(A + B)$ , for positive semidefinite matrices  $A, B$ , with  $U$  a unitary matrix that appears in the polar decomposition of  $BA$ , is also extended, using some known norm inequalities, associated to Furuta inequality and Araki-Cordes inequality.

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**Title:** Log-majorizations for the eigenvalues of the Cartan barycenter

**Speaker:** Yongdo Lim (ylim@skku.edu), Sungkyunkwan University, Suwon, Korea.

**Co-authors:** Fumio Hiai.

**Abstract:** We present several log-majorizations for the (symplectic) eigenvalues of the Cartan barycenter of integrable probability Borel measures on the Riemannian manifold of positive definite matrices. This leads a version of Jensen's inequality for geometric integrals of matrix-valued integrable random variables.

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**Title:** The ray nonsingularity of certain uniformly random ray patterns

**Speaker:** Yue Liu (liuyue@fzu.edu.cn), Fuzhou University, Fuzhou, China.

**Abstract:** A uniformly random ray pattern matrix  $A$  with a given zero-nonzero pattern (described by a digraph  $D$  with no multi-arcs or loops) is the matrix whose nonzero entries are mutually independent random variables uniformly distributed over the unit circle  $\mathbf{S}^1$  in the complex plane. It is shown in this paper that the probability of  $I - A$  to be ray nonsingular is completely determined by the cycle graph  $CG(D)$  of  $D$  (i.e. the adjacency structure of the directed cycles in  $D$ ) if  $CG(D)$  is a tree. A formula is given to compute the probability when  $CG(D)$  is a tree, and it is also shown that as the order of  $CG(D)$  tends to infinity, the limit of the probability is 0.

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**Title:** Semipositivity of matrices and linear maps relative to proper cones

**Speaker:** Vatsalkumar N. Mer (vatsl.n15@iisertvm.ac.in), Indian Institute of Science Education and Research - Thiruvananthapuram, Vithura, Thiruvananthapuram, Kerala, India.

**Co-authors:** Schindranath Jayaraman (sachindranathj@iisertvm.ac.in), Indian Institute of Science Education and Research - Thiruvananthapuram, Vithura, Thiruvananthapuram, Kerala, India and Chandrashekar Arumugasamy (chandrashekar@cutn.ac.in), School of Mathematics and Computer Science, Central University of Tamilnadu, Thiruvarur, Tamilnadu, India.

**Abstract:** For a proper cone  $K$  in a finite dimensional real Hilbert space  $V$ , a square matrix  $A$  (a linear map  $L$ ) is said to be  $K$ -semipositive if there exists  $d \in K^\circ$ , the interior of  $K$ , such that  $Ad \in K^\circ$  ( $Ld \in K^\circ$ ). Our aim is to characterize  $K$ -semipositivity of matrices / linear maps relative to a proper cone  $K$ . Among various results obtained, we prove

1. a decomposition result for  $K$ -semipositive matrices in the form  $YX^{-1}$ , where  $Y$  and  $X$  are  $K$ -positive ( $A(K \setminus \{0\}) \subseteq K^\circ$ ) with  $X$  invertible.
2. an explicit decomposition for semipositive matrices over the Lorentz cone.
3. characterize semipositivity of the maps  $X \mapsto AXB$ ,  $X \mapsto AXB + B^t X A^t$  over the cone  $\mathcal{S}_+^n$  of positive semidefinite matrices in the space of real symmetric matrices  $\mathcal{S}^n$ , where  $A, B \in M_n(\mathbb{R})$ .
4. the semipositive cone of a matrix  $A$  and its invariance under  $A$ .
5. a characterization of nonnegativity relative to two proper cones.

These are based on results in [1] and [2] below.

## References

- [1] A. Chandrashekar, Sachindranath Jayaraman and Vatsalkumar N. Mer, *A characterization of nonnegativity relative to proper cones*, <https://arxiv.org/abs/1801.09849> (Preliminary version).
- [2] A. Chandrashekar, Sachindranath Jayaraman and Vatsalkumar N. Mer, *Semipositivity of matrices and linear maps relative to proper cones*, [http://www.optimization-online.org/DB\\_HTML/2018/02/6478.html](http://www.optimization-online.org/DB_HTML/2018/02/6478.html).

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**Title:** An improved voting power index based on relations between the players.

**Speaker:** Dr. Himadri Mukherjee (himadrim@goa.bits-pilani.ac.in), Birla Institute of Technology and Sciences-Pilani, Goa Campus, Goa, India.

**Abstract:** Voting power indices are an important part of game theory with a history of fervent research by many eminent mathematicians of the last century. Among the many proposed power indices, Banzhaf index is endowed with many applications in fields ranging from economics to search engine algorithms. Since the inception, the power indices have been generalized in many directions, keeping in view both theoretical and applicability aspects of the index. In this talk, we present an improved index that will take into account the voting games, in which players have a certain power of persuasion on other players. Given a set of players  $\{p_1, p_1, \dots, p_n\}$ , real number  $\alpha_{i,j}$  encodes the persuasion power of the player  $p_i$  on the player  $p_j$  in a vote. The matrix  $A = (\alpha_{i,j})_{n \times n}$  is called the association matrix of the voting game. This improvement has many applications and much closer to the reality. This improvement brings in the rich field of linear algebra into the purely combinatorial definition of the voting game thus making it not only more applicable but also much more interesting to the mathematical community. In a recent paper we have described a computational method of the improved index and an application of the improved index in the voting of the Council of the European Union, in this paper we demonstrate more theoretical linear algebraic results for the improved index. We have obtained a few interesting inequalities between the index with association matrix and without an association matrix. We have further proved a result for an association matrix that is diagonalizable and described the Banzhaf indices. We have also listed a few future research directions in this field.

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**Title:** Unitary equivalence classes of some types of quantum walks

**Speaker:** Hiromichi Ohno (h\_ohno@shinshu-u.ac.jp), Shinshu University, Nagano, Japan.

**Abstract:** Quantum walks can be considered as a quantum analog of classical random walks and have been studied in various fields, such as quantum information theory and quantum probability theory. A quantum walk is defined by a pair  $(U, \{\mathcal{H}_v\}_{v \in V})$ , in which  $V$  is a countable set,  $\{\mathcal{H}_v\}_{v \in V}$  is a family of separable Hilbert spaces, and  $U$  is a unitary operator on  $\mathcal{H} = \bigoplus_{v \in V} \mathcal{H}_v$ . In this talk, we discuss unitary equivalence classes of two-dimensional two-state quantum walks and quantum walks on a circle.

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**Title:** Loss of orthogonality in large sparse matrix computations

**Speaker:** Christopher Paige (Paige@cs.mcgill.ca), McGill University, Montreal, Canada.

**Abstract:** Many large sparse matrix algorithms are based on orthogonality, but for efficiency this orthogonality is often obtained via short term recurrences. This can lead to both loss of orthogonality and loss of linear independence of computed vectors, yet with well designed algorithms high accuracy can still be obtained. Here we discuss a nice theoretical indicator of loss of orthogonality and linear independence, and for such short term recurrence algorithms show how it can lead to a related higher dimensional orthogonality that can be used to analyze and prove the effectiveness of such algorithms. We illustrate advantages and shortcomings of such algorithms with Cornelius Lanczos' Hermitian matrix tridiagonalization process.

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**Title:** The subspaces spanned by Householder vectors associated with an orthogonal or a symplectic matrix

**Speaker:** Agnes T. Paras (agnes@math.upd.edu.ph), University of the Philippines Diliman, Quezon City, Philippines.

**Co-authors:** Kennett L. Dela Rosa (University of the Philippines Diliman) and Dennis I. Merino (Southeastern Louisiana University)

**Abstract:** The Cartan-Dieudonné-Scherk Theorem guarantees that every orthogonal matrix can be written as a product of matrices of the form  $H_u \equiv I - uu^T$ , where  $u \in \mathbb{C}^n$  satisfies  $u^T u = 2$ ; moreover every symplectic matrix can be written as a product of matrices of the form  $H_u \equiv$

$I - uu^T J$ , where  $u$  is nonzero and  $J = \begin{bmatrix} 0 & I \\ -I & 0 \end{bmatrix}$ . If  $\mathcal{V}$  is a nonempty subset of  $\mathbb{C}^n$  and  $S = I$  or  $J$ , we define the  $S$ -orthogonal complement of  $\mathcal{V}$  to be

$$\mathcal{V}^S = \{x \in \mathbb{C}^n \mid v^T Sx = 0 \text{ for all } v \in \mathcal{V}\}.$$

If  $Q = H_{u_1} H_{u_2} \cdots H_{u_r}$ , set  $\mathcal{U} = \text{span}\{u_1, \dots, u_r\}$ . We study the relationship between  $Q$ ,  $\mathcal{U}$  and  $\text{Im}(Q - I)$ . Suppose that  $r$  is minimal. We show that if  $\dim \mathcal{U} = r$ , then  $\text{Im}(Q - I) = \mathcal{U}$ . Furthermore,  $S(Q - I)$  is not skew-symmetric if and only if  $\dim \mathcal{U} = r$ . If  $\mathcal{W} = \text{Im}(Q - I)$ , we show that a relationship between  $\mathcal{W}$  and  $\mathcal{W}^S$  determines the Jordan structure of  $Q$ .

**Title:** The affine property of quasi-free states on self-dual CAR algebras

**Speaker:** Yusuke Sawada (m14017c@math.nagoya-u.ac.jp), Nagoya University, Nagoya, Japan

**Abstract:** In quantum field theory, particles according to Fermi statistics are described by a set of operators that satisfy canonical anticommutation relations. The  $C^*$ -algebra consisting of such operators is called a CAR algebra  $A$  and states of the quantum system are described by linear functionals on  $A$ . There is a one-to-one correspondence between quasi-free states on a self-dual CAR algebra and covariance operators. In this talk, we introduce a necessary and sufficient condition for that the correspondence preserves convex combinations in the case when Hilbert spaces which we consider have finite dimensions and covariance operators (matrix) commute.

**Title:** Generalized bilinear form graphs over finite commutative rings

**Speaker:** Siripong Sirisuk (siripong.srs@gmail.com), Chulalongkorn University, Bangkok, Thailand.

**Co-author:** Yotsanan Meemark (yotsanan.m@chula.ac.th), Chulalongkorn University, Bangkok, Thailand.

**Abstract:** Let  $R$  be a finite commutative ring with identity,  $n \in \mathbb{N}$  and  $\beta$  a bilinear form on  $R^n$ . We count the numbers of free submodules and totally isotropic free submodules of  $R^n$  of rank  $s$  by using the lifting idea. We apply the rank of matrices over commutative rings defined by McCoy to define the graph whose vertex set is the set of totally isotropic free submodules of  $R^n$  of rank  $s$  called the *generalized bilinear form graph*. We study this graph when  $(R^n, \beta)$  is a symplectic space. We show that we can decompose the graph over a finite commutative ring into the tensor products of graphs over finite local rings.

**Title:** The generalized numerical range of a set of matrices

**Speaker:** Raymond Nung-Sing Sze (raymond.sze@polyu.edu.hk), The Hong Kong Polytechnic University, Hong Kong

**Co-authors:** P.S. Lau (HK PolyU), C.K. Li (William & Mary), Y.T. Poon (Iowa State U)

**Abstract:** For a given set of  $n \times n$  matrices  $\mathcal{F}$ , we study the union of the  $C$ -numerical ranges of the matrices in the set  $\mathcal{F}$ , denoted by  $W_C(\mathcal{F})$ . In this talk, we present some basic algebraic and topological properties of  $W_C(\mathcal{F})$ , and show that there are connections between the geometric properties of  $W_C(\mathcal{F})$  and the algebraic properties of  $C$  and the matrices in  $\mathcal{F}$ . Furthermore, we consider the starshapedness and convexity of the set  $W_C(\mathcal{F})$ . In particular, we show that if  $\mathcal{F}$  is the convex hull of two matrices such that  $W_C(A)$  and  $W_C(B)$  are convex, then the set  $W_C(\mathcal{F})$  is star-shaped. We also investigate the extensions of the results to the joint  $C$ -numerical range of an  $m$ -tuple of matrices.

**Title:** Some Nonlinear Problems in Linear Algebra

**Speaker:** Tin-Yau Tam (ttam@unr.edu, tamtiny@auburn.edu), University of Nevada-Reno and Auburn University, USA

**Abstract:** We will discuss some problems in Linear Algebra including Aluthge iteration and its convergence, Marcus-de Oliveira Conjecture on the determinants of sums of two normal matrices

with prescribed eigenvalues, and Thompson's inequalities on singular values and diagonal entries of a complex symmetric matrix.

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**Title:** Operator means and generalized relative entropy.

**Speaker:** Yukihiro Tsurumi (ra0006hk@ed.ritsumei.ac.jp), Ritsumeikan University, Kusatsu, Japan.

**Co-authors:** Hiroyuki Osaka (Ritsumeikan University) and Shuhei Wada (National Institute of Technology, Kisarazu College).

**Abstract:** It is well known that the operator mean is related to quantum information theory. In this talk we present a relation between generalized relative entropy and operator means. The generalized relative entropy is what we formulated the relative entropy Furuichi introduced in 2012 ([J. Inequal. Appl. 2012, 2012:226, 16 pp.]). We also talk about our results for the operator means. Let  $\sigma$  be an operator mean in the sense of Kubo-Ando and let  $\nabla_\alpha$  be a weighted arithmetic mean. If  $\text{Tr}(A\sigma B) \geq \text{Tr}(A\nabla_\alpha B - \max\{\alpha, 1-\alpha\}|A-B|)$  holds for all positive semidefinite matrices  $A, B$ , then there exists  $\beta \in [0, 1]$  such that  $\sigma = \nabla_\beta$ .

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**Title:** An integral representation of some operator monotone functions

**Speaker:** Yoichi Udagawa (yoi-uda@fc.ritsumei.ac.jp), Ritsumeikan University, Japan.

**Abstract:** We call  $f(z)$  a Pick function if  $f(z)$  is holomorphic on  $\mathbb{C}^+ = \{z \in \mathbb{C} \mid \Im z > 0\}$  and satisfies  $f(\mathbb{C}^+) \subset \mathbb{C}^+$ . By Löwner's results, an operator monotone function  $f(x)$  defined on an interval  $(a, b)$  has an analytic continuation to the upper half plane as a Pick function, and, conversely, if a Pick function  $f(z)$  satisfies  $f((a, b)) \subset \mathbb{R}$  for an interval  $(a, b)$ , then the restriction of  $f(z)$  to  $(a, b)$  is operator monotone. Moreover, it is well-known that a Pick function  $f(z)$  has an integral representation

$$f(z) = \alpha z + \beta + \int_{-\infty}^{\infty} \left( \frac{1}{\lambda - z} - \frac{\lambda}{\lambda^2 + 1} \right) d\mu(\lambda)$$

where  $\alpha \geq 0$ ,  $\beta \in \mathbb{R}$  and  $\mu(\lambda)$  is a nonnegative Borel measure on  $\mathbb{R}$ .

We obtain an integral representation of holomorphic function  $P_\alpha(z)$  which is real on the positive part of the real axis and formed

$$P_\alpha(x) = \left( \frac{x^\alpha + 1}{2} \right)^{\frac{1}{\alpha}} \quad (x > 0).$$

For this purpose we define a two variable function which is substituted for an argument  $\theta$ , and also find an explicit real and imaginary part of  $P_\alpha(x + iy)$ .

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**Title:** Some inequalities for matrix Heinz mean

**Speaker:** Bich-Khue T. Vo (bksphcm@gmail.com), University of Finance Marketing, Ho Chi Minh city, Viet Nam.

**Co-authors:** Trung Hoa Dinh (trunghoa.math@gmail.com), University of North Florida, 1 UNF Dr, Jacksonville, FL 32224, USA, and Tin-Yau Tam (ttam@unr.edu), University of Nevada, Reno, USA.

**Abstract:** In this talk, we prove some reverse norm inequalities for the Heinz mean and for the Hilbert-Schmidt norm. A new characterization of the arithmetic mean is also obtained.

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**Title:** Commutation matrices and commutation tensors

**Speaker:** Changqing Xu (cqxurichard@usts.edu.cn), Suzhou University of Science and Technology, Suzhou, China.

**Co-author:** Zhibing Chen, Shenzhen University

**Abstract:** The commutation matrices were initialized in statistics where they are also called permutation matrices. Actually a commutation matrix  $K_{p,q} = [A_{ij}]$  is defined as an  $p \times q$  block

matrix whose  $(i, j)$  block  $A_{ij} \in R^{q \times p}$  is an  $q \times p$  matrix whose entries are all zero except the  $(j, i)$  entry which is 1. The commutation matrix can be used to establish the relationship of a matrix and its transpose, but this is achieved through the vectorization. In this talk, we will introduce the commutation tensor, and employ it to directly express the transpose of a matrix as a multilinear transformation of itself. We will also define operations such as the multiplications and the power of commutation tensors, and investigate some interesting properties of commutation tensors. We will also employ our results on commutation tensors to obtain a uniform linear preserver of determinant.

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**Title:** On numerical range of a generalization of the Aluthge transform.

**Speaker:** Takeaki Yamazaki (t-yamazaki@toyo.jp), Toyo University, Kawagoe, Japan.

**Abstract:** Let  $T = U|T|$  be a polar decomposition of a bounded linear operator on a complex Hilbert space. The Aluthge transform is defined by  $\Delta(T) := |T|^{\frac{1}{2}}U|T|^{\frac{1}{2}}$ . In the recent research, S. H. Lee, W. Y. Lee and J. Yoon defined the mean transform as  $\hat{T} := \frac{U|T|+|T|U}{2}$ . In this talk, we shall generalize the above operator transforms. In fact, we will introduce a generalization of the Aluthge transform in the viewpoint of the operator means. It contains the Aluthge and the mean transforms. Then we shall introduce a property of numerical range of a generalization of the Aluthge transformation.

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**Title:** Enumerating vertices of the polytopes of stochastic tensors by an optimization approach

**Speaker:** Fuzhen Zhang (zhang@nova.edu), Nova Southeastern University, Florida, USA

**Abstract:** We are concerned with the extreme points of the polytopes of stochastic tensors. By a tensor we mean a multi-dimensional array over the real number field. A line-stochastic tensor is a nonnegative tensor in which the sum of all entries on each line (i.e., 1 free index) is equal to 1; a plane-stochastic tensor is a nonnegative tensor in which the sum of all entries on each plane (i.e., 2 free indices) is equal to 1. In enumerating extreme points of the polytopes of line- and plane-stochastic tensors of order 3 and dimension  $n$ , we consider the approach by linear optimization and present new lower and upper bounds. We also study the coefficient matrices that define the polytopes. (Joint work with Xiao-Dong Zhang.)

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**Title:** Maple package for matrix computations

**Speaker:** Yang Zhang (yang.zhang@umanitoba.ca), University of Manitoba, Winnipeg, Canada.

**Abstract:** In this talk, we introduce a Maple package for computing some questions in matrices over quaternions and skew polynomials, which include computing various generalized inverses and solving matrix equations. Some algorithms and examples are presented.

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