問題 4.4.1

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(1) $\mathbf{a} = \left(\dfrac{-y}{x^2+y^2}, \dfrac{x}{x^2+y^2}, z \right)$, $\mathbf{r}(t) = (\cos \pi t, \sin \pi t, t)$ について

$$\mathbf{a}(\mathbf{r}(t)) = (-\sin \pi t, \cos \pi t, t)$$

$$\dot{\mathbf{r}}(t) = (-\pi \sin \pi t, \pi \cos \pi t, 1)$$

$$\therefore \int_C \mathbf{a} \cdot d\mathbf{r} = \int_0^1 \mathbf{a}(\mathbf{r}(t)) \cdot \dot{\mathbf{r}}(t)\, dt$$

$$= \int_0^1 \left\{ \pi (\sin^2 \pi t + \cos^2 \pi t) + t \right\} dt$$

$$= \int_0^1 (\pi + t)\, dt = \left[\pi t + \frac{t^2}{2} \right]_0^1$$

$$= \pi + \frac{1}{2}$$

(2) $\mathbf{a} = (xz^2, yz^2, z)$, $\mathbf{r}(t) = (t^2, t^2, t)$ について

$$\mathbf{a}(\mathbf{r}(t)) = (t^4, t^4, t)$$

$$\dot{\mathbf{r}}(t) = (2t, 2t, 1)$$

$$\therefore \int_C \mathbf{a} \cdot d\mathbf{r} = \int_0^1 \mathbf{a}(\mathbf{r}(t)) \cdot \dot{\mathbf{r}}(t)\, dt$$

$$= \int_0^1 (2t^5 + 2t^5 + t)\, dt$$

$$= \int_0^1 (4t^5 + t)\, dt = \left[\frac{4t^6}{6} + \frac{t^2}{2} \right]_0^1$$

$$= \frac{2}{3} + \frac{1}{2} = \frac{7}{6}$$

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