3.8節

1.(1) $\hat{f}(u)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(t)e^{-iut}dt=\frac{1}{\sqrt{2\pi}}\int_{-4}^{4}(x-4)e^{-iut}dt$

$=\frac{1}{\sqrt{2\pi}}\left(\left[-\frac{1}{iu}(x-4)e^{-iut}\right]_{-4}^{4}+\frac{1}{iu}\int_{-4}^{4}e^{-iut}dt\right)$

$=\frac{1}{\sqrt{2\pi}}\left(\frac{8i}{u}e^{4iu}+\left[\frac{1}{u^2}e^{-iut}\right]_{-4}^{4}\right)$

$=\frac{1}{\sqrt{2\pi}}\left(\frac{8i}{u}e^{4iu}+\frac{1}{u^2}e^{-4iu}+\frac{1}{u^2}e^{4iu}\right)$ となる

(2) $\hat{f}(u)=\frac{1}{\sqrt{2\pi}}\int_{-1}^{1}|x|e^{-iut}dt=\frac{1}{\sqrt{2\pi}}\left(\int_{-1}^{0}-xe^{-iut}dt+\int_{0}^{1}xe^{-iut}dt\right)$

$=\frac{1}{\sqrt{2\pi}}\left(\left[\frac{1}{iu}xe^{-iut}\right]_{-1}^{0}-\frac{1}{iu}\int_{-1}^{0}e^{-iut}dt+\left[-\frac{1}{iu}xe^{-iut}\right]_{0}^{1}+\frac{1}{iu}\int_{0}^{1}e^{-iut}dt\right)$

$=\frac{1}{\sqrt{2\pi}}\left(-\frac{i}{u}e^{iu}-\frac{1}{u^2}\left[e^{-iut}\right]_{-1}^{0}+\frac{i}{u}e^{-iu}+\frac{1}{u^2}\left[e^{-iut}\right]_{0}^{1}\right)$

$=\frac{1}{\sqrt{2\pi}}\left(-\frac{i}{u}e^{iu}-\frac{2}{u^2}+\frac{1}{u^2}e^{iu}+\frac{i}{u}e^{-iu}+\frac{1}{u^2}e^{-iu}\right)$ である.

(3) $\hat{f}(u)=\frac{1}{\sqrt{2\pi}}\int_{0}^{\infty}e^{-t}e^{-iut}dt=\frac{1}{\sqrt{2\pi}}\int_{0}^{\infty}e^{-(1+iu)t}dt$

$=\frac{1}{\sqrt{2\pi}}\left[-\frac{1}{1+iu}e^{-(1+iu)t}\right]_{0}^{\infty}=\frac{1}{\sqrt{2\pi}}\cdot\frac{1}{1+iu}$ である

(4) $\hat{f}(u)=\frac{1}{\sqrt{2\pi}}\int_{0}^{1}x^3e^{-iut}dt=\frac{1}{\sqrt{2\pi}}\left(\left[-\frac{1}{iu}x^3e^{-iut}\right]_{0}^{1}+\frac{3}{iu}\int_{0}^{1}x^2e^{-iut}dt\right)$

$=\frac{1}{\sqrt{2\pi}}\left(\frac{1}{u}e^{-iu}+\left[\frac{3}{u^2}x^2e^{-iut}\right]_{0}^{1}-\frac{6}{u^2}\int_{0}^{1}xe^{-iut}dt\right)$

$=\frac{1}{\sqrt{2\pi}}\left(\frac{1}{u}e^{-iu}+\frac{3}{u^2}e^{-iu}+\frac{6}{iu^3}\left[xe^{-iut}\right]_{0}^{1}-\frac{6}{iu^3}\int_{0}^{1}e^{-iut}dt\right)$

No.

$$=\frac{1}{\sqrt{2\pi}}\left(\frac{1}{u}e^{-iu}+\frac{3}{u^2}e^{-iu}-\frac{6i}{u^3}e^{-iu}-\frac{6}{u^4}\left[e^{-iut}\right]_0^1\right)$$

$$=\frac{1}{\sqrt{2\pi}}\left(\frac{1}{u}e^{-iu}+\frac{3}{u^2}e^{-iu}-\frac{6i}{u^3}e^{-iu}-\frac{6}{u^4}e^{-iu}+\frac{6}{u^4}\right)$$ である

2(1) $\hat{f}(u)=\frac{1}{\sqrt{2\pi}}\int_{-2}^{2}2e^{-iut}dt=\frac{1}{\sqrt{2\pi}}\left[-\frac{2}{iu}e^{-iut}\right]_{-2}^{2}$

$$=\frac{1}{\sqrt{2\pi}}\cdot\frac{2i}{u}\left(e^{-2iu}-e^{2iu}\right)$$ である. また.

$$f(x)\sim\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\hat{f}(u)e^{iux}du=\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{2i}{u}\left(e^{-2iu}-e^{2iu}\right)e^{iux}du$$

$$=\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{2}{u}\frac{e^{2iu}-e^{-2iu}}{2i}\cdot(\cos ux+i\sin ux)\,du$$

$$=\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{2}{u}\cdot\sin 2u\,(\cos ux+i\sin ux)\,du$$

$$=\frac{4}{\pi}\int_{0}^{\infty}\frac{1}{u}\sin 2u\cdot\cos ux\,du$$ となる.

(2) $\hat{f}(u)=\frac{1}{\sqrt{2\pi}}\int_{-1}^{1}te^{-iut}dt=\frac{1}{\sqrt{2\pi}}\left(\left[-\frac{1}{iu}te^{-iut}\right]_{-1}^{1}+\frac{1}{iu}\int_{-1}^{1}e^{-iut}dt\right)$

$$=\frac{1}{\sqrt{2\pi}}\left(\frac{i}{u}\left(e^{-iu}+e^{iu}\right)+\frac{1}{u^2}\left[e^{-iut}\right]_{-1}^{1}\right)$$

$$=\frac{1}{\sqrt{2\pi}}\left(\frac{i}{u}\left(e^{-iu}+e^{iu}\right)+\frac{1}{u^2}\left(e^{-iu}-e^{iu}\right)\right)$$ である. また.

$$f(x)\sim\frac{1}{2\pi}\int_{-\infty}^{\infty}\left(\frac{i}{u}\left(e^{-iu}+e^{iu}\right)+\frac{1}{u^2}\left(e^{-iu}-e^{iu}\right)\right)e^{iux}du$$

$$=\frac{1}{\pi}\int_{-\infty}^{\infty}\left(\frac{i}{u}\cdot\cos u-\frac{i}{u^2}\sin u\right)(\cos ux+i\sin ux)\,du$$

$$= \frac{2}{\pi}\int_0^\infty \left(-\frac{1}{u}\cos u + \frac{1}{u^2}\sin u\right)\sin ux \, du$$ となる．

(3) $$\hat{f}(u) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{-|t|} e^{-iut}\, dt = \frac{1}{\sqrt{2\pi}}\left(\int_{-\infty}^{0} e^{(1-iu)t}\, dt + \int_0^\infty e^{(-1-iu)t}\, dt\right)$$

$$= \frac{1}{\sqrt{2\pi}}\left(\left[\frac{1}{1-iu}e^{(1-iu)t}\right]_{-\infty}^{0} + \left[\frac{-1}{1+iu}e^{-(1+iu)t}\right]_0^\infty\right)$$

$$= \frac{1}{\sqrt{2\pi}}\left(\frac{1}{1-iu} + \frac{1}{1+iu}\right) = \frac{1}{\sqrt{2\pi}}\cdot\frac{2}{1+u^2}$$ である．また．

$$f(x) \sim \frac{1}{\pi}\int_{-\infty}^{\infty}\frac{1}{1+u^2}e^{iux}\, du = \frac{1}{\pi}\int_{-\infty}^{\infty}\frac{1}{1+u^2}(\cos ux + i\sin ux)\, du$$

$$= \frac{1}{\pi}\int_0^\infty \frac{2}{1+u^2}\cos ux\, du$$ となる．

(4) $$\hat{f}(u) = \frac{1}{\sqrt{2\pi}}\int_{-3}^{3}(t^2-1)e^{-iut}\, dt$$

$$= \frac{1}{\sqrt{2\pi}}\left(\left[-\frac{1}{iu}(t^2-1)e^{-iut}\right]_{-3}^{3} + \frac{2}{iu}\int_{-3}^{3} t e^{-iut}\, dt\right)$$

$$= \frac{1}{\sqrt{2\pi}}\left(\frac{i}{u}\left(8e^{-3iu} - 8e^{3iu}\right) + \left[\frac{2}{u^2}te^{-iut}\right]_{-3}^{3} - \frac{2}{u^2}\int_{-3}^{3}e^{-iut}\, dt\right)$$

$$= \frac{1}{\sqrt{2\pi}}\left(\frac{8i}{u}(e^{-3iu} - e^{3iu}) + \frac{6}{u^2}(e^{-3iu} + e^{3iu}) + \frac{2}{iu^3}\left[e^{-iut}\right]_{-3}^{3}\right)$$

$$= \frac{1}{\sqrt{2\pi}}\left(\frac{8i}{u}(e^{-3iu} - e^{3iu}) + \frac{6}{u^2}(e^{-3iu} + e^{3iu}) - \frac{2i}{u^3}(e^{-3iu} - e^{3iu})\right)$$ となる．また

$$f(x) \sim \frac{1}{2\pi}\int_{-\infty}^{\infty}\left(\frac{16}{u}\sin 3u + \frac{12}{u^2}\cos 3u - \frac{4}{u^3}\sin 3u\right)(\cos ux + i\sin ux)\, du$$

$$= \frac{1}{\pi}\int_0^\infty \left(\frac{16}{u}\sin 3u + \frac{12}{u^2}\cos 3u - \frac{4}{u^3}\sin 3u\right)\cos ux\, du .$$