No.

3.7節

1 (1) $C_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\,dx = \frac{1}{2\pi}\int_0^{\pi} 2\,dx = 1$

$C_n = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x) e^{-inx} dx = \frac{1}{2\pi}\int_0^{\pi} 2 e^{-inx} dx = \frac{1}{\pi}\left[-\frac{1}{in} e^{-inx}\right]_0^{\pi}$

$= \frac{i}{\pi n}((-1)^n - 1)$ より

$f(x) \sim 1 + \sum_{n\neq 0} \frac{i}{\pi n}((-1)^n - 1) e^{inx}$

(2) $C_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} |x|\,dx = \frac{1}{\pi}\int_0^{\pi} x\,dx = \frac{1}{2}\pi$

$C_n = \frac{1}{2\pi}\int_{-\pi}^{\pi} |x| e^{-inx} dx = \frac{1}{2\pi}\int_{-\pi}^{0} -x e^{-inx} dx + \frac{1}{2\pi}\int_0^{\pi} x e^{-inx} dx$

$= \frac{1}{2\pi}\left[\frac{1}{in} x e^{-inx}\right]_{-\pi}^{0} - \int_{-\pi}^{0} \frac{1}{2\pi i n} e^{-inx} dx$

$+ \frac{1}{2\pi}\left[-\frac{1}{in} x e^{-inx}\right]_0^{\pi} + \frac{1}{2\pi i n}\int_0^{\pi} e^{-inx} dx$

$= \frac{1}{2\pi i n}\pi(-1)^n - \left[\frac{1}{2\pi n^2} e^{-inx}\right]_{-\pi}^{0} - \frac{1}{2\pi i n}\pi(-1)^n + \frac{1}{2\pi n^2}\left[e^{-inx}\right]_0^{\pi}$

$= \frac{1}{2\pi n^2}(-1 + (-1)^n + (-1)^n - 1) = \frac{1}{\pi n^2}((-1)^n - 1)$

$\therefore f(x) \sim \frac{1}{2}\pi + \sum_{n\neq 0} \frac{1}{\pi n^2}((-1)^n - 1) e^{inx}$

(3) $\sin 3x = \frac{e^{3ix} - e^{-3ix}}{2i}$ より $f(x) \sim \frac{1}{2i}(e^{3ix} - e^{-3ix})$

(4) $C_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} x^3\,dx = 0$

$C_n = \frac{1}{2\pi}\int_{-\pi}^{\pi} x^3 e^{-inx} dx = \frac{1}{2\pi}\left[-\frac{1}{in} x^3 e^{-inx}\right]_{-\pi}^{\pi} + \frac{3}{2\pi i n}\int_{-\pi}^{\pi} x^2 e^{-inx} dx$

No.

$$= \frac{i}{2\pi n}\left(\pi^3(-1)^n-(-\pi)^3(-1)^n\right)+\frac{3}{2\pi n^2}\left[x^2e^{-inx}\right]_{-\pi}^{\pi}-\frac{3}{\pi n^2}\int_{-\pi}^{\pi}xe^{-inx}dx$$

$$= \frac{i\pi^2}{n}(-1)^n+\frac{3}{i\pi n^3}\left[xe^{-inx}\right]_{-\pi}^{\pi}-\frac{3}{i\pi n^3}\int_{-\pi}^{\pi}e^{-inx}dx$$

$$= \frac{i\pi^2}{n}(-1)^n-\frac{3i}{\pi n^3}\left(\pi(-1)^n+\pi(-1)^n\right)-\frac{3}{\pi n^4}\left[e^{-inx}\right]_{-\pi}^{\pi}$$

$$= \frac{i\pi^2}{n}(-1)^n-\frac{6i}{n^3}(-1)^n$$ より

$$f(x)\sim\sum_{n\neq 0}\left(\frac{i\pi^2}{n}(-1)^n-\frac{6i}{n^3}(-1)^n\right)e^{inx}$$

2(1) $C_0=\frac{1}{2\pi}\int_{-\pi}^{\pi}x\,dx=0.$

$$C_n=\frac{1}{2\pi}\int_{-\pi}^{\pi}xe^{-inx}dx=\frac{1}{2\pi}\left[-\frac{1}{in}xe^{-inx}\right]_{-\pi}^{\pi}+\frac{1}{2\pi in}\int_{-\pi}^{\pi}e^{-inx}dx$$

$$= \frac{i}{2\pi n}\left(\pi(-1)^n+\pi(-1)^n\right)+\frac{1}{2\pi n^2}\left[e^{-inx}\right]_{-\pi}^{\pi}=\frac{i}{n}(-1)^n$$ より

$$f(x)\sim\sum_{n\neq 0}\frac{i}{n}(-1)^n e^{inx}.$$ である. また.

$$f(x)\sim\sum_{n=1}^{\infty}\frac{i}{n}(-1)^n e^{inx}+\sum_{n=1}^{\infty}\frac{i}{(-n)}(-1)^{(-n)}\cdot e^{i(-n)x}$$

$$= \sum_{n=1}^{\infty}\frac{i}{n}(-1)^n\cdot\left(e^{inx}-e^{-inx}\right)$$

$$= \sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}2\cdot\frac{e^{inx}-e^{-inx}}{2i}=\sum_{n=1}^{\infty}\frac{2}{n}(-1)^{n+1}\sin nx$$ となる.

(2) $C_0=\frac{1}{2\pi}\int_{-\pi}^{\pi} x^2-x+1\,dx=\frac{1}{\pi}\int_0^{\pi}x^2+1\,dx=\frac{1}{\pi}\left[\frac{1}{3}x^3+x\right]_0^{\pi}=\frac{\pi^2}{3}+1.$

$$C_n=\frac{1}{2\pi}\int_{-\pi}^{\pi}(x^2-x+1)e^{-inx}dx=\frac{1}{2\pi}\left[-\frac{1}{in}(x^2-x+1)e^{-inx}\right]_{-\pi}^{\pi}+\frac{1}{2\pi in}\int_{-\pi}^{\pi}(2x-1)e^{-inx}dx$$

$$=\frac{i}{2\pi n}(-1)^n\left((\pi^2-\pi+1)-(\pi^2+\pi+1)\right)$$

$$+\frac{1}{2\pi n^2}\left[(2x-1)e^{-inx}\right]_{-\pi}^{\pi}-\frac{1}{\pi n^2}\int_{-\pi}^{\pi}e^{-inx}dx$$

$$=\frac{i}{n}(-1)^{n+1}+\frac{1}{2\pi n^2}(-1)^n\left(2\pi-1-(-2\pi-1)\right)+\frac{1}{\pi i n^3}\left[e^{-inx}\right]_{-\pi}^{\pi}$$

$=\frac{i}{n}(-1)^{n+1}+\frac{2}{n^2}(-1)^n$ より

$f(x)\sim 1+\frac{\pi^2}{3}+\sum_{n\neq 0}\left(\frac{i}{n}(-1)^{n+1}+\frac{2}{n^2}(-1)^n\right)e^{inx}$ である. また

$$f(x)\sim 1+\frac{\pi^2}{3}+\sum_{n=1}^{\infty}\left(\frac{i}{n}(-1)^{n+1}+\frac{2}{n^2}(-1)^n\right)e^{inx}+\sum_{n=1}^{\infty}\left(\frac{i}{-n}(-1)^{n+1}+\frac{2}{n^2}(-1)^n\right)e^{-inx}$$

$$=1+\frac{\pi^2}{3}+\sum_{n=1}^{\infty}\frac{4}{n^2}(-1)^n\cdot\frac{e^{inx}+e^{-inx}}{2}+\frac{2}{n}(-1)^n\frac{e^{inx}-e^{-inx}}{2i}$$

$=1+\frac{\pi^2}{3}+\sum_{n=1}^{\infty}\frac{4}{n^2}(-1)^n\cos nx+\frac{2}{n}(-1)^n\sin nx$ である.

(3) $f(x)-1=g(x)$ とすると. $g(x)=\begin{cases}0 & -\pi<x<0\\ x & 0\leqq x\leqq \pi\end{cases}$ である.

$g(x)$ のフーリエ級数は.

$C_0=\frac{1}{2\pi}\int_0^{\pi}x\,dx=\frac{1}{4}\pi.$

$$C_n=\frac{1}{2\pi}\int_0^{\pi}xe^{-inx}dx=\frac{1}{2\pi}\left[-\frac{1}{in}xe^{-inx}\right]_0^{\pi}+\frac{1}{2\pi in}\int_0^{\pi}e^{-inx}dx$$

No.

$$= \frac{i}{2n}(-1)^n + \frac{1}{2\pi n^2}\left[e^{-inx}\right]_0^{\pi} = \frac{i}{2n}(-1)^n + \frac{1}{2\pi n^2}\left((-1)^n - 1\right) \quad \text{より}$$

$$g(x) \sim \frac{1}{4}\pi + \sum_{n \neq 0}\left(\frac{i}{2n}(-1)^n + \frac{1}{2\pi n^2}((-1)^n - 1)\right)e^{inx}$$

$$\therefore f(x) \sim 1 + \frac{1}{4}\pi + \sum_{n \neq 0}\left(\frac{i}{2n}(-1)^n + \frac{1}{2\pi n^2}((-1)^n - 1)\right)e^{inx} \quad \text{である. また}$$

$$f(x) \sim 1 + \frac{1}{4}\pi + \sum_{n=1}^{\infty}\left(\frac{i}{2n}(-1)^n + \frac{1}{2\pi n^2}((-1)^n - 1)\right)e^{inx} + \sum_{n=1}^{\infty}\left(-\frac{i}{2n}(-1)^n + \frac{1}{2\pi n^2}((-1)^n - 1)\right)e^{-inx}$$

$$= 1 + \frac{1}{4}\pi + \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \cdot \frac{e^{inx} - e^{-inx}}{2i} + \frac{1}{\pi n^2}((-1)^n - 1)\frac{e^{inx} + e^{-inx}}{2}$$

$$= 1 + \frac{1}{4}\pi + \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \sin nx + \frac{1}{\pi n^2}((-1)^n - 1)\cos nx \quad \text{である.}$$

(4) $$C_n = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^x e^{-inx}\, dx = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{(1-in)x}\, dx$$

$$= \frac{1}{2\pi} \cdot \frac{1}{1-in}\left[e^{(1-in)x}\right]_{-\pi}^{\pi} = \frac{1}{2\pi} \cdot \frac{1}{1-in}\left(e^{\pi - in\pi} - e^{-\pi + in\pi}\right)$$

$$= \frac{1}{2\pi(1-in)}(-1)^n \cdot (e^{\pi} - e^{-\pi}) \quad \text{より}$$

$$f(x) \sim \sum \frac{1}{2\pi(1-in)}(-1)^n(e^{\pi} - e^{-\pi})e^{inx} \quad \text{となる. また,}$$

$$f(x) \sim \frac{1}{\pi}(e^{\pi} - e^{-\pi})\left(\sum \frac{1+in}{2(1+n^2)}(-1)^n e^{inx}\right)$$

$$= \frac{1}{\pi}(e^{\pi} - e^{-\pi})\left(\frac{1}{2} + \sum_{n=1}^{\infty}\frac{1+in}{2(1+n^2)}(-1)^n e^{inx} + \sum_{n=1}^{\infty}\frac{1-in}{2(1+n^2)}(-1)^n e^{-inx}\right)$$

$$= \frac{1}{\pi}(e^{\pi} - e^{-\pi})\left(\frac{1}{2} + \sum_{n=1}^{\infty}\frac{(-1)^n}{1+n^2} \cdot \frac{e^{inx} + e^{-inx}}{2} - \frac{n}{1+n^2}(-1)^n \cdot \frac{e^{inx} - e^{-inx}}{2i}\right)$$

$$= \frac{1}{\pi}(e^{\pi} - e^{-\pi})\left(\frac{1}{2} + \sum_{n=1}^{\infty}\frac{(-1)^n}{n^2+1}\cos nx - \frac{n}{n^2+1}(-1)^n \sin nx\right) \quad \text{である.}$$