No.

3.6節

1 (1) 余弦 $C(u)=\sqrt{\frac{2}{\pi}}\int_0^{\infty} f(t)\cos ut\,dt=\sqrt{\frac{2}{\pi}}\int_0^{a}\cos ut\,dt$

$=\sqrt{\frac{2}{\pi}}\cdot\left[\frac{1}{u}\sin ut\right]_0^a=\sqrt{\frac{2}{\pi}}\cdot\frac{1}{u}\sin au$ となる

正弦 $S(u)=\sqrt{\frac{2}{\pi}}\int_0^{\infty} f(t)\cos ut\,dt=\sqrt{\frac{2}{\pi}}\int_0^{a}\sin ut\,dt$

$=\sqrt{\frac{2}{\pi}}\left[-\frac{1}{u}\cos ut\right]_0^a=\sqrt{\frac{2}{\pi}}\left(-\frac{1}{u}\cos au+\frac{1}{u}\right)$ である.

(2) $C(u)=\sqrt{\frac{2}{\pi}}\int_0^a (a-t)\cos ut\,dt=\sqrt{\frac{2}{\pi}}\left(\left[\frac{1}{u}(a-t)\sin ut\right]_0^a+\frac{1}{u}\int_0^a\sin ut\,dt\right)$

$=\sqrt{\frac{2}{\pi}}\cdot\frac{1}{u}\left[-\frac{1}{u}\cos ut\right]_0^a=\sqrt{\frac{2}{\pi}}\frac{1}{u^2}(1-\cos au)$

$S(u)=\sqrt{\frac{2}{\pi}}\int_0^a(a-t)\sin ut\,dt=\sqrt{\frac{2}{\pi}}\left(\left[-\frac{1}{u}(a-t)\cos ut\right]_0^a-\frac{1}{u}\int_0^a\cos ut\,dt\right)$

$=\sqrt{\frac{2}{\pi}}\left(\frac{a}{u}-\frac{1}{u}\left[\frac{1}{u}\sin ut\right]_0^a\right)=\sqrt{\frac{2}{\pi}}\left(\frac{a}{u}-\frac{1}{u^2}\sin au\right)$ である.

(3) $C(u)=\sqrt{\frac{2}{\pi}}\int_0^3(4t-5)\cos ut\,dt=\sqrt{\frac{2}{\pi}}\left(\left[\frac{1}{u}(4t-5)\sin ut\right]_0^3-\frac{4}{u}\int_0^3\sin ut\,dt\right)$

$=\sqrt{\frac{2}{\pi}}\left(\frac{7}{u}\sin 3u-\frac{4}{u}\left[-\frac{1}{u}\cos ut\right]_0^3\right)$

$=\sqrt{\frac{2}{\pi}}\left(\frac{7}{u}\sin 3u+\frac{4}{u^2}(\cos 3u-1)\right)$

$S(u)=\sqrt{\frac{2}{\pi}}\int_0^3(4t-5)\sin ut\,dt=\sqrt{\frac{2}{\pi}}\left(\left[-\frac{1}{u}(4t-5)\cos ut\right]_0^3+\frac{4}{u}\int_0^3\cos ut\,dt\right)$

$=\sqrt{\frac{2}{\pi}}\left(-\frac{7}{u}\cos 3u-\frac{5}{u}+\frac{4}{u}\left[\frac{1}{u}\sin ut\right]_0^3\right)$

$=\sqrt{\frac{2}{\pi}}\left(-\frac{7}{u}\cos 3u-\frac{5}{u}+\frac{4}{u^2}\sin 3u\right)$ である

(4) $C(u)=\sqrt{\frac{2}{\pi}}\int_0^1 (3t^2+2t-1)\cos ut\,dt$

$=\sqrt{\frac{2}{\pi}}\left(\left[\frac{1}{u}(3t^2+2t-1)\sin ut\right]_0^1-\frac{1}{u}\int_0^1(6t+2)\sin ut\,dt\right)$

$=\sqrt{\frac{2}{\pi}}\left(\frac{4}{u}\sin u-\frac{1}{u}\left[-\frac{1}{u}(6t+2)\cos ut\right]_0^1-\frac{6}{u^2}\int_0^1\cos ut\,dt\right)$

$=\sqrt{\frac{2}{\pi}}\left(\frac{4}{u}\sin u+\frac{8}{u^2}\cos u-\frac{2}{u^2}-\frac{6}{u^2}\left[\frac{1}{u}\sin ut\right]_0^1\right)$

$=\sqrt{\frac{2}{\pi}}\left(\frac{4}{u}\sin u+\frac{8}{u^2}\cos u-\frac{2}{u^2}-\frac{6}{u^3}\sin u\right)$

$S(u)=\sqrt{\frac{2}{\pi}}\int_0^1 (3t^2+2t-1)\sin ut\,dt$

$=\sqrt{\frac{2}{\pi}}\left(\left[-\frac{1}{u}(3t^2+2t-1)\cos ut\right]_0^1+\frac{1}{u}\int_0^1(6t+2)\cos ut\,dt\right)$

$=\sqrt{\frac{2}{\pi}}\left(-\frac{4}{u}\cos u-\frac{1}{u}+\frac{1}{u}\left[\frac{1}{u}(6t+2)\sin ut\right]_0^1-\frac{6}{u^2}\int_0^1\sin ut\,dt\right)$

$=\sqrt{\frac{2}{\pi}}\left(-\frac{4}{u}\cos u-\frac{1}{u}+\frac{8}{u^2}\sin u-\frac{6}{u^2}\left[-\frac{1}{u}\cos ut\right]_0^1\right)$

$=\sqrt{\frac{2}{\pi}}\left(-\frac{4}{u}\cos u-\frac{1}{u}+\frac{8}{u^2}\sin u+\frac{6}{u^3}\cos u-\frac{6}{u^3}\right)$ となる

(5) $a\leqq 1$ のとき.

$C(u)=\sqrt{\frac{2}{\pi}}\int_0^a (1-t)\cos ut\,dt=\sqrt{\frac{2}{\pi}}\left(\left[\frac{1}{u}(1-t)\sin ut\right]_0^a+\frac{1}{u}\int_0^a\sin ut\,dt\right)$

$=\sqrt{\frac{2}{\pi}}\left(\frac{1}{u}(1-a)\sin au+\frac{1}{u}\left[-\frac{1}{u}\cos ut\right]_0^a\right)$

$=\sqrt{\frac{2}{\pi}}\left(\frac{1}{u}(1-a)\sin au-\frac{1}{u^2}(\cos au-1)\right)$

$$S(u)=\sqrt{\frac{2}{\pi}}\int_0^a (1-t)\sin ut\,dt=\sqrt{\frac{2}{\pi}}\left(\left[-\frac{1}{u}(1-t)\cos ut\right]_0^a-\frac{1}{u}\int_0^a \cos ut\,dt\right)$$

$$=\sqrt{\frac{2}{\pi}}\left(\frac{1}{u}(a-1)\cos au+\frac{1}{u}-\frac{1}{u}\left[\frac{1}{u}\sin ut\right]_0^a\right)$$

$$=\sqrt{\frac{2}{\pi}}\left(\frac{1}{u}(a-1)\cos au+\frac{1}{u}-\frac{1}{u^2}\sin au\right)$$

である. また $a\geqq 1$ のときは.

$$C(u)=\sqrt{\frac{2}{\pi}}\int_0^a f(t)\cos ut\,dt=\sqrt{\frac{2}{\pi}}\int_0^1 (1-t)\cos ut\,dt+\sqrt{\frac{2}{\pi}}\int_1^a (t-1)\cos ut\,dt$$

$$=\sqrt{\frac{2}{\pi}}\left(\frac{1}{u^2}(1-\cos u)+\int_1^a (t-1)\cos ut\,dt\right)$$

← 上の計算で $a=1$ としたものを使った.

$$=\sqrt{\frac{2}{\pi}}\left(\frac{1}{u^2}(1-\cos u)+\left[\frac{1}{u}(t-1)\sin ut\right]_1^a-\frac{1}{u}\int_1^a \sin ut\,dt\right)$$

$$=\sqrt{\frac{2}{\pi}}\left(\frac{1}{u^2}(1-\cos u)+\frac{1}{u}(a-1)\sin au-\frac{1}{u}\left[-\frac{1}{u}\cos ut\right]_1^a\right)$$

$$=\sqrt{\frac{2}{\pi}}\left(\frac{1}{u^2}-\frac{2}{u^2}\cos u+\frac{1}{u}(a-1)\sin au+\frac{1}{u^2}\cos au\right)$$

$$S(u)=\sqrt{\frac{2}{\pi}}\int_0^1 (1-t)\sin ut\,dt+\sqrt{\frac{2}{\pi}}\int_1^a (t-1)\sin ut\,dt$$

$$=\sqrt{\frac{2}{\pi}}\left(\frac{1}{u}-\frac{1}{u^2}\sin u+\left[-\frac{1}{u}(t-1)\cos ut\right]_1^a+\frac{1}{u}\int_1^a \cos ut\,dt\right)$$

$$=\sqrt{\frac{2}{\pi}}\left(\frac{1}{u}-\frac{1}{u^2}\sin u-\frac{1}{u}(a-1)\cos au+\frac{1}{u}\left[\frac{1}{u}\sin ut\right]_1^a\right)$$

$$=\sqrt{\frac{2}{\pi}}\left(\frac{1}{u}-\frac{2}{u^2}\sin u-\frac{1}{u}(a-1)\cos au+\frac{1}{u^2}\sin au\right)$$ である

No.

(6) $A=\int_0^a e^{2t}\cos ut\,dt$ とすると.

$$A=\left[\frac{1}{u}e^{2t}\sin ut\right]_0^a-\frac{2}{u}\int_0^a e^{2t}\sin ut\,dt$$

$$=\frac{1}{u}e^{2a}\sin au-\frac{2}{u}\left[-\frac{1}{u}e^{2t}\cos ut\right]_0^a-\frac{4}{u^2}\int_0^a e^{2t}\cos ut\,dt$$

$$=\frac{1}{u}e^{2a}\sin au+\frac{2}{u^2}e^{2a}\cos au-\frac{2}{u^2}-\frac{4}{u^2}A$$

$\therefore A=\dfrac{1}{u^2+4}\left(ue^{2a}\sin au+2e^{2a}\cos au-2\right)$ となり.

$C(u)=\dfrac{\sqrt{2}}{\sqrt{\pi}(u^2+4)}\left(ue^{2a}\sin au+2e^{2a}\cos au-2\right)$ となる.

また上の計算から.

$$\int_0^a e^{2t}\sin ut=\frac{u}{2}\left(-A+\frac{1}{u}e^{2a}\sin au\right)$$

$$=\frac{1}{2}\cdot\frac{1}{u^2+4}\left(-u^2e^{2a}\sin au-2ue^{2a}\cos au+2u+(u^2+4)e^{2a}\sin au\right)$$

$=\dfrac{1}{u^2+4}\left(2e^{2a}\sin au-ue^{2a}\cos au+u\right)$ となり.

$S(u)=\dfrac{\sqrt{2}}{\sqrt{\pi}}\dfrac{1}{u^2+4}\left(2e^{2a}\sin au-ue^{2a}\cos au+u\right)$ となる.

2. $C(u)=\sqrt{\dfrac{2}{\pi}}\displaystyle\int_0^\pi \sin t\cdot\cos ut\,dt=\sqrt{\dfrac{2}{\pi}}\cdot\dfrac{1}{2}\int_0^\pi \sin(1+u)t+\sin(1-u)t\,dt$

$$=\frac{1}{\sqrt{2\pi}}\left[-\frac{1}{1+u}\cos(1+u)t+\frac{1}{u-1}\cos(u-1)t\right]_0^\pi$$

$=\dfrac{1}{\sqrt{2\pi}}\left(-\dfrac{1}{1+u}+\dfrac{1}{u-1}\right)\left(\cos(1+u)\pi-1\right)=\sqrt{\dfrac{2}{\pi}}\dfrac{1}{1-u^2}\left(\cos\pi u+1\right)$ より

No.

$$f(x) \sim \frac{2}{\pi}\int_0^\infty \frac{1}{1-u^2}(\cos\pi u+1)\cos ux\,du$$ となり.

$$\int_0^\infty \frac{1}{1-u^2}(\cos\pi u+1)\cos ux\,du = \begin{cases} \frac{\pi}{2}|\sin x| & (|x| \leqq \pi) \\ 0 & \text{その他} \end{cases}$$ となる

3. $A = \int_0^\infty e^{-t}\sin ut\,dt$ とすると

$$A = \left[-e^{-t}\sin ut\right]_0^\infty + u\int_0^\infty e^{-t}\cos ut\,dt$$

$$= \left[-ue^{-t}\cos ut\right]_0^\infty - u^2\int_0^\infty e^{-t}\sin ut\,dt$$

$$= u - u^2 A$$ より $A = \frac{u}{u^2+1}$ である これより

$$f(x) \sim \frac{2}{\pi}\int_0^\infty \frac{u}{u^2+1}\sin ux\,du$$ となり

$$\int_0^\infty \frac{u}{u^2+1}\sin ux = \frac{\pi}{2}e^{-x}$$ を得る.