3.5節

1.(1) $\int_{-l}^{l} |1|\, dx = 2l \longrightarrow \infty \quad (l \to \infty)$ より絶対積分不可能.

(2) $l = 2\pi n + m \quad$ (nは自然数, $0 \leqq m < 2\pi$) とすると

$$\int_{-l}^{l} |\sin 2x|\, dx \geqq \int_{-2\pi n}^{2\pi n} |\sin 2x|\, dx = 4n \cdot \int_0^{\frac{\pi}{2}} \sin 2x\, dx$$

$|\sin 2x|$ は周期 $\frac{\pi}{2}$ なので.

$= 4n \cdot \left[-\frac{1}{2}\cos 2x\right]_0^{\frac{\pi}{2}} = 4n \longrightarrow \infty \quad (n \to \infty)$ より絶対積分不可能.

(3). $\int_{-l}^{l} |x e^{-x^2}| dx = 2 \cdot \int_0^l x e^{-x^2} dx = 2 \cdot \left[-\frac{1}{2} e^{-x^2}\right]_0^l$

$= -e^{-l^2} + 1 \longrightarrow 1 \quad (l \to \infty)$ より絶対積分可能

(4) $\int_{\frac{1}{l}}^{l} \frac{1}{x^2}\, dx = \left[-\frac{1}{x}\right]_{\frac{1}{l}}^{l} = -\frac{1}{l} + l \longrightarrow \infty \quad (l \to \infty)$

より絶対積分不可能.

2(1) $f(x)$は偶関数より. $B(u) = 0$.

$A(u) = \int_{-\infty}^{\infty} f(t) \cdot \cos ut\, dt = 2 \cdot \int_0^1 \cos ut\, dt = 2\left[\frac{1}{u}\sin ut\right]_0^1$

$= \frac{2}{u}\sin u$.

$\therefore f(x) \sim \frac{1}{\pi}\int_0^{\infty} \frac{2}{u}\sin u \cos ux\, du$.

(2). $f(x)$は奇関数より $A(u) = 0$.

$B(u) = \int_{-\infty}^{\infty} f(t)\sin ut\, dt = 2\int_0^2 x\sin ut\, dt = \left[-\frac{2}{u}x\cos ut\right]_0^2 + \frac{2}{u}\int_0^2 \cos ut\, dt$

No.

$$= -\frac{4}{u}\cos 2u + \frac{2}{u}\left[\frac{1}{u}\sin ut\right]_0^2 = -\frac{4}{u}\cos 2u + \frac{2}{u^2}\sin 2u$$

$$\therefore f(x) \sim \frac{1}{\pi}\int_0^\infty \left(-\frac{4}{u}\cos 2u + \frac{2}{u^2}\sin 2u\right)\sin ux\,du$$

(3) $A(u) = \int_0^2 (3-2t)\cos ut\,dt = \left[\frac{1}{u}(3-2t)\sin ut\right]_0^2 + \frac{2}{u}\int_0^2 \sin ut\,dt$

$$= -\frac{1}{u}\sin 2u + \frac{2}{u}\left[-\frac{1}{u}\cos ut\right]_0^2 = -\frac{1}{u}\sin 2u + \frac{2}{u^2}(-\cos 2u + 1)$$

$B(u) = \int_0^2 (3-2t)\sin ut\,dt = \left[-\frac{1}{u}(3-2t)\cos ut\right]_0^2 - \frac{2}{u}\int_0^2 \cos ut\,dt$

$$= \frac{1}{u}\cos 2u + \frac{3}{u} - \frac{2}{u}\left[\frac{1}{u}\sin ut\right]_0^2 = \frac{1}{u}\cos 2u + \frac{3}{u} - \frac{2}{u^2}\sin 2u$$ より

$$f(x) \sim \frac{1}{\pi}\int_0^\infty \left(-\frac{1}{u}\sin 2u + \frac{2}{u^2}(-\cos 2u + 1)\right)\cos ux + \left(\frac{1}{u}\cos 2u + \frac{3}{u} - \frac{2}{u^2}\sin 2u\right)\sin ux\,du$$

である

(4) $f(x)$ は偶関数より. $B(u) = 0$

$A(u) = 2\int_0^\pi (\pi - t)\cos ut\,dt = 2\left[(\pi - t)\cdot\frac{1}{u}\sin ut\right]_0^\pi + \frac{2}{u}\int_0^\pi \sin ut\,dt$

$$= \frac{2}{u}\left[-\frac{1}{u}\cos ut\right]_0^\pi = -\frac{2}{u^2}(\cos \pi u - 1)$$ より

$$f(x) \sim \frac{1}{\pi}\int_0^\infty \frac{2}{u^2}(1 - \cos \pi u)\cos ux\,du$$ である.

(5). $A(u) = \int_{-1}^2 (t^2 - 3)\cos ut\,dt = \left[\frac{1}{u}(t^2-3)\sin ut\right]_{-1}^2 - \frac{2}{u}\int_{-1}^2 t\sin ut\,dt$

$$= \frac{1}{u}(\sin 2u - 2\sin u) - \frac{2}{u}\left[t\cdot -\frac{1}{u}\cos ut\right]_{-1}^2 - \frac{2}{u^2}\int_{-1}^2 \cos ut\,dt$$

$$= \frac{1}{u}(\sin 2u - 2\sin u) + \frac{2}{u^2}(2\cos 2u + \cos u) - \frac{2}{u^2}\left[\frac{1}{u}\sin ut\right]_{-1}^{2}$$

$$= \frac{1}{u}(\sin 2u - 2\sin u) + \frac{2}{u^2}(2\cos 2u + \cos u) - \frac{2}{u^3}(\sin 2u + \sin u)$$

$$B(u) = \int_{-1}^{2}(t^2-3)\sin ut\,dt = \left[-\frac{1}{u}(t^2-3)\cos ut\right]_{-1}^{2} + \frac{2}{u}\int_{-1}^{2} t\cos ut\,dt$$

$$= -\frac{1}{u}(\cos 2u + 2\cos u) + \frac{2}{u}\left[\frac{1}{u}t\sin ut\right]_{-1}^{2} - \frac{2}{u^2}\int_{-1}^{2}\sin ut\,dt$$

$$= -\frac{1}{u}(\cos 2u + 2\cos u) + \frac{2}{u^2}(2\sin 2u - \sin u) - \frac{2}{u^2}\left[-\frac{1}{u}\cos ut\right]_{-1}^{2}$$

$$= -\frac{1}{u}(\cos 2u + 2\cos u) + \frac{2}{u^2}(2\sin 2u - \sin u) + \frac{2}{u^3}(\cos 2u - \cos u)$$ より

$$f(x) \sim \frac{1}{\pi}\int_0^\infty \left\{\frac{1}{u}(\sin 2u - 2\sin u) + \frac{2}{u^2}(2\cos 2u + \cos u) - \frac{2}{u^3}(\sin 2u + \sin u)\right\}\cos ux$$
$$+ \left\{-\frac{1}{u}(\cos 2u - 2\cos u) + \frac{2}{u^2}(2\sin 2u - \sin u) + \frac{2}{u^3}(\cos 2u - \cos u)\right\}\sin ux\,du$$

である

(6) $f(x)$は奇関数より　$A(u)=0$

$$B(u) = 2\int_0^1 t^3\sin ut\,dt = 2\cdot\left[-\frac{1}{u}t^3\cos ut\right]_0^1 + \frac{6}{u}\int_0^1 t^2\cos ut\,dt$$

$$= -\frac{2}{u}\cos u + \frac{6}{u}\left[\frac{1}{u}t^2\sin ut\right]_0^1 - \frac{12}{u^2}\int_0^1 t\sin ut\,dt$$

$$= -\frac{2}{u}\cos u + \frac{6}{u^2}\sin u - \frac{12}{u^2}\left[-\frac{1}{u}t\cos ut\right]_0^1 - \frac{12}{u^3}\int_0^1 \cos ut\,dt$$

$$= -\frac{2}{u}\cos u + \frac{6}{u^2}\sin u + \frac{12}{u^3}\cos u - \frac{12}{u^3}\left[\frac{1}{u}\sin ut\right]_0^1$$

$$= -\frac{2}{u}\cos u + \frac{6}{u^2}\sin u + \frac{12}{u^3}\cos u - \frac{12}{u^4}\sin u$$

$$f(x) \sim \frac{1}{\pi}\int_0^\infty \left(-\frac{2}{u}\cos u + \frac{6}{u^2}\sin u + \frac{12}{u^3}\cos u - \frac{12}{u^4}\sin u\right)\sin ux\,du.$$