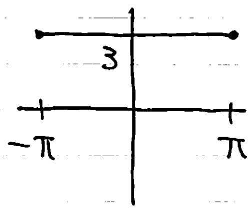
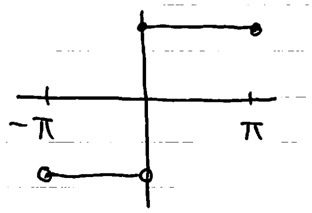
3.2 節

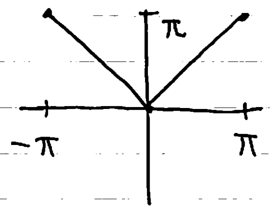
1. $f(-x)=f(x)$ $\qquad$ $f(-x)=-f(x)$

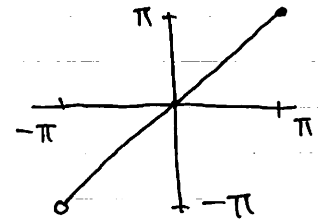
(1)



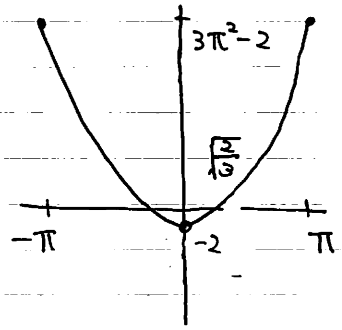


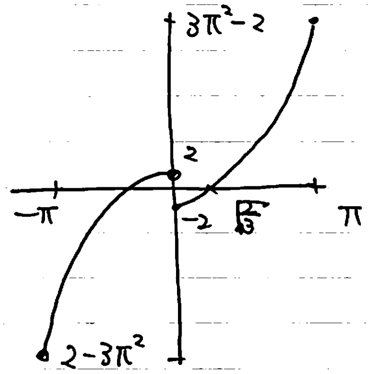
(2)



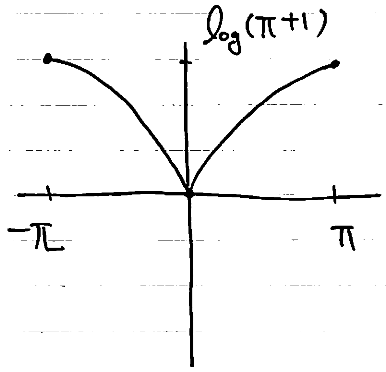


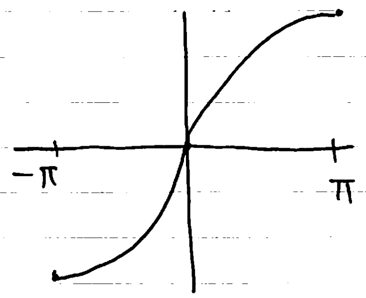
(3)





(4)





2(1) 余弦 $\quad a_0=\frac{2}{\pi}\int_0^{\pi}\pi-x\,dx=\frac{2}{\pi}\left[\pi x-\frac{1}{2}x^2\right]_0^{\pi}=\frac{2}{\pi}\left(\pi^2-\frac{1}{2}\pi^2\right)=\pi.$

$$a_n=\frac{2}{\pi}\int_0^{\pi}(\pi-x)\cos nx\,dx=\frac{2}{\pi}\left[\frac{1}{n}(\pi-x)\sin nx\right]_0^{\pi}+\frac{2}{\pi n}\int_0^{\pi}\sin nx\,dx$$

$$=\frac{2}{\pi n}\left[-\frac{1}{n}\cos nx\right]_0^{\pi}=-\frac{2}{\pi n^2}\left((-1)^n-1\right)=\frac{2}{\pi n^2}\left(1-(-1)^n\right)$$

$$\therefore f(x) \sim \frac{\pi}{2} + \sum_{n=1}^{\infty} \frac{2}{\pi n^2}\left(1-(-1)^n\right)\cos nx$$

正弦

$$b_n = \frac{2}{\pi}\int_0^{\pi}(\pi - x)\sin nx\,dx = \frac{2}{\pi}\left[-\frac{1}{n}(\pi - x)\cos nx\right]_0^{\pi} - \frac{2}{\pi n}\int_0^{\pi}\cos nx\,dx$$

$$= \frac{2}{n} - \frac{2}{\pi n}\left[\frac{1}{n}\sin nx\right]_0^{\pi} = \frac{2}{n}$$ より

$$f(x) \sim \sum_{n=1}^{\infty}\frac{2}{n}\sin nx$$

(2) 余弦 $a_0 = \frac{2}{\pi}\int_0^{\pi} 1\,dx = 2$, $a_n = \frac{2}{\pi}\int_0^{\pi}\cos nx\,dx = \left[\frac{2}{\pi n}\sin nx\right]_0^{\pi} = 0$

$$\therefore f(x) \sim 1$$

正弦

$$b_n = \frac{2}{\pi}\int_0^{\pi}\sin nx\,dx = \frac{2}{\pi}\left[-\frac{1}{n}\cos nx\right]_0^{\pi} = \frac{2}{\pi n}\left(1-(-1)^n\right)$$

$$\therefore f(x) \sim \sum_{n=1}^{\infty}\frac{2}{\pi n}\left(1-(-1)^n\right)\sin nx$$

(3) 余弦 $a_0 = \frac{2}{\pi}\int_0^{\pi} -x-2\,dx = \frac{2}{\pi}\left[-\frac{1}{2}x^2 - 2x\right]_0^{\pi} = \frac{2}{\pi}\left(-\frac{1}{2}\pi^2 - 2\pi\right)$

$$= -\pi - 4$$

$$a_n = \frac{2}{\pi}\int_0^{\pi}(-x-2)\cos nx\,dx = \frac{2}{\pi}\left[\frac{1}{n}(-x-2)\sin nx\right]_0^{\pi} + \frac{2}{\pi n}\int_0^{\pi}\sin nx\,dx$$

$$= \frac{2}{\pi n}\left[-\frac{1}{n}\cos nx\right]_0^{\pi} = \frac{1}{\pi n^2}\left(1-(-1)^n\right)$$

$$\therefore f(x) \sim -\frac{1}{2}\pi - 2 + \sum_{n=1}^{\infty}\frac{1}{\pi n^2}\left(1-(-1)^n\right)\cos nx$$

正弦 $b_n = \frac{2}{\pi}\int_0^{\pi}(-x-2)\sin nx\,dx = \frac{2}{\pi}\left[\frac{1}{n}(x+2)\cos nx\right]_0^{\pi} - \frac{2}{\pi n}\int_0^{\pi}\cos nx\,dx$

$$=\frac{2}{\pi n}\left((\pi+2)(-1)^n-2\right)-\frac{2}{\pi n}\left[\frac{1}{n}\sin nx\right]_0^\pi$$ より

$$f(x)\sim\sum_{n=1}^{\infty}\frac{2}{\pi n}\left((\pi+2)(-1)^n-2\right)\sin nx.$$ である.

(4) 余弦 $a_0=\frac{2}{\pi}\int_0^\pi x^2+4x\,dx=\frac{2}{\pi}\left[\frac{1}{3}x^3+2x^2\right]_0^\pi=\frac{2}{\pi}\left(\frac{1}{3}\pi^3+2\pi^2\right)=\frac{2}{3}\pi^2+4\pi$

$$a_n=\frac{2}{\pi}\int_0^\pi(x^2+4x)\cos nx\,dx=\frac{2}{\pi}\left[\frac{1}{n}(x^2+4x)\sin nx\right]_0^\pi-\frac{2}{\pi n}\int_0^\pi(2x+4)\sin nx\,dx$$

$$=-\frac{2}{\pi n}\left[-\frac{1}{n}(2x+4)\cos nx\right]_0^\pi-\frac{4}{\pi n^2}\int_0^\pi\cos nx\,dx$$

$$=\frac{2}{\pi n^2}\left((2\pi+4)(-1)^n-4\right)-\left[\frac{4}{\pi n^3}\sin nx\right]_0^\pi=\frac{4}{\pi n^2}\left(-2+(\pi+2)(-1)^n\right)$$ より

$$f(x)\sim\frac{1}{3}\pi^2+2\pi+\sum_{n=1}^{\infty}\frac{4}{\pi n^2}\left(-2+(\pi+2)(-1)^n\right)\cos nx$$ である.

正弦 $b_n=\frac{2}{\pi}\int_0^\pi(x^2+4x)\sin nx\,dx=\frac{2}{\pi}\left[-\frac{1}{n}(x^2+4x)\cos nx\right]_0^\pi+\frac{2}{\pi n}\int_0^\pi(2x+4)\cos nx\,dx$

$$=\frac{-2}{\pi n}\left((\pi^2+4\pi)(-1)^n\right)+\frac{2}{\pi n}\left[\frac{1}{n}(2x+4)\sin nx\right]_0^\pi-\frac{4}{\pi n^2}\int_0^\pi\sin nx\,dx$$

$$=\frac{2}{\pi n}(\pi^2+4\pi)(-1)^{n+1}-\frac{4}{\pi n^2}\left[-\frac{1}{n}\cos nx\right]_0^\pi$$

$$=\frac{2}{\pi n}(\pi^2+4\pi)(-1)^{n+1}+\frac{4}{\pi n^3}\left((-1)^n-1\right)$$ より

$$f(x)\sim\sum_{n=1}^{\infty}\left(\frac{2}{\pi n}(\pi^2+4\pi)(-1)^{n+1}+\frac{4}{\pi n^3}\left((-1)^n-1\right)\right)\sin nx$$ である.

(5) 余弦 $a_0=\frac{2}{\pi}\int_0^\pi\sin x\,dx=\frac{2}{\pi}\left[-\cos x\right]_0^\pi=\frac{4}{\pi}$

$$a_1=\frac{2}{\pi}\int_0^\pi\sin x\cos x\,dx=\frac{1}{\pi}\int_0^\pi\sin 2x\,dx=\left[\frac{1}{2\pi}\cos 2x\right]_0^\pi=0$$

No.

$$a_n=\frac{2}{\pi}\int_0^\pi \sin x\cos nx\,dx=\frac{1}{\pi}\int_0^\pi \sin(n+1)x-\sin(n-1)x\ dx$$

$$=\frac{1}{\pi}\left[-\frac{1}{n+1}\cos(n+1)x+\frac{1}{n-1}\cos(n-1)x\right]_0^\pi$$

$$=\frac{1}{\pi}\left(-\frac{1}{n+1}((-1)^{n+1}-1)+\frac{1}{n-1}((-1)^{n-1}-1)\right)=\frac{1}{\pi}((-1)^{n+1}-1)+\frac{2}{n^2-1}$$

$$=\frac{2}{\pi(n^2-1)}((-1)^{n+1}-1)$$ より

$$f(x)\sim\frac{2}{\pi}+\sum_{n=2}^{\infty}\frac{2}{\pi(n^2-1)}((-1)^{n+1}-1)\cos nx$$ である.

正弦 $f(x)\sim\sin x$.

(6) 余弦 $a_0=\frac{2}{\pi}\int_0^\pi e^{2x}dx=\left[\frac{1}{\pi}e^{2x}\right]_0^\pi=\frac{1}{\pi}(e^{2\pi}-1)$

$A=\int_0^\pi e^{2x}\cos nx\,dx$ とすると.

$$A=\left[\frac{1}{n}e^{2x}\sin nx\right]_0^\pi-\frac{2}{n}\int_0^\pi e^{2x}\sin nx\,dx$$

$$=-\frac{2}{n}\left[e^{2x}\left(-\frac{1}{n}\cos nx\right)\right]_0^\pi-\frac{4}{n^2}\int e^{2x}\cos nx\,dx$$

$$=\frac{2}{n^2}\left(e^{2\pi}(-1)^n-1\right)-\frac{4}{n^2}A \qquad \therefore A=\frac{2}{n^2+4}\left(e^{2\pi}(-1)^n-1\right)$$

$$\therefore a_n=\frac{2}{\pi}A=\frac{4}{\pi(n^2+4)}\left(e^{2\pi}(-1)^n-1\right)$$

$$\therefore f(x)\sim\frac{1}{2\pi}(e^{2\pi}-1)+\sum_{n=1}^{\infty}\frac{4}{\pi(n^2+4)}\left(e^{2\pi}(-1)^n-1\right)\cos nx.$$

正弦. Aの計算から.

$$b_n=\frac{2}{\pi}\int_0^\pi e^{2x}\sin nx\,dx=-\frac{n}{\pi}A=\frac{2n}{\pi(n^2+4)}\left(1-e^{2\pi}(-1)^n\right)$$

$$\therefore f(x)\sim\sum_{n=1}^{\infty}\frac{2n}{\pi(n^2+4)}\left(1-e^{2\pi}(-1)^n\right)$$