問題1.2.1

$\boxed{1}$ (1) $x\frac{dy}{dx} = -(y+1)$ ∴ $\frac{dy}{y+1} = -\frac{dx}{x}$ ∴ $\log(y+1) = -\log x + c = \log\frac{e^c}{x}$

∴ $y+1 = \frac{e^c}{x}$ ∴ $y = \frac{c}{x} - 1$

(2) $(y+1)\frac{dy}{dx} = 1-x$ ∴ $(y+1)dy = (1-x)dx$ ∴ $\frac{1}{2}(y+1)^2 = x - \frac{x^2}{2} + c$

∴ $(y+1)^2 = 2x - x^2 + 2c$ ∴ $(y+1)^2 = 2x - x^2 + c$

(3) $\frac{dy}{dx} = (\tan y)(\tan x)$ ∴ $\frac{dy}{\tan y} = \tan x\,dx$ ∴ $\frac{\cos y}{\sin y}dy = \frac{\sin x}{\cos x}dx$

∴ $\int\frac{\cos y}{\sin y}dy = \int\frac{\sin x}{\cos x}dx + c$ ∴ $\log\sin y = -\log\cos x + c = \log\frac{e^c}{\cos x}$

∴ $\sin y = \frac{e^c}{\cos x}$ ∴ $\sin y = \frac{c}{\cos x}$

(4) $xy(1+x^2)\frac{dy}{dx} = 1+y^2$ ∴ $\frac{y}{1+y^2}dy = \frac{dx}{x(1+x^2)}$

∴ $\frac{1}{2}\log(1+y^2) = \int\left(\frac{1}{x} - \frac{x}{1+x^2}\right)dx + c = \log x - \frac{1}{2}\log(1+x^2) + c = \log\frac{e^c\cdot x}{\sqrt{1+x^2}}$

∴ $\log\sqrt{1+y^2} = \log\frac{e^c\cdot x}{\sqrt{1+x^2}}$ ∴ $\sqrt{1+y^2} = \frac{e^c\cdot x}{\sqrt{1+x^2}}$ ∴ $1+y^2 = \frac{(e^c)^2x^2}{1+x^2}$

∴ $(1+x^2)(1+y^2) = e^{2c}\cdot x^2$ ∴ $(1+x^2)(1+y^2) = cx^2$

(5) $(1+x)y + 2(1-y)x\frac{dy}{dx} = 0$ ∴ $2(y-1)x\frac{dy}{dx} = (x+1)y$

∴ $\frac{y-1}{y}dy = \frac{x+1}{2x}dx$ ∴ $\left(1-\frac{1}{y}\right)dy = \frac{1}{2}\left(1+\frac{1}{x}\right)dx$

∴ $\int\left(1-\frac{1}{y}\right)dy = \frac{1}{2}\int\left(1+\frac{1}{x}\right)dx + c$ ∴ $y - \log y = \frac{1}{2}(x + \log x) + c$

∴ $2y - 2\log y = x + \log x + 2c$ ∴ $\log xy^2 = 2y - x - 2c$ ∴ $xy^2 = e^{2y-x-2c}$

∴ $xy^2 = e^{-2c}\cdot e^{2y-x}$ ∴ $xy^2 = ce^{2y-x}$

(6) $y\frac{dy}{dx} = xe^{x^2}\cdot e^{y^2}$ ∴ $ye^{-y^2}dy = xe^{x^2}dx$ ∴ $\int ye^{-y^2}dy = \int xe^{x^2}dx + c$

∴ $-\frac{e^{-y^2}}{2} = \frac{e^{x^2}}{2} + c$ ∴ $-e^{-y^2} = e^{x^2} + 2c$ ∴ $e^{x^2} + e^{-y^2} = c$

(7) $(1-x^2)\frac{dy}{dx}=y^2-1$ ∴ $\frac{dy}{y^2-1}=-\frac{dx}{x^2-1}$

∴ $\frac{1}{2}\int\left(\frac{1}{y-1}-\frac{1}{y+1}\right)dy=-\frac{1}{2}\int\left(\frac{1}{x-1}-\frac{1}{x+1}\right)dx+c$

∴ $\frac{1}{2}\log\frac{y-1}{y+1}=-\frac{1}{2}\log\frac{x-1}{x+1}+c$ ∴ $\log\frac{y-1}{y+1}=\log e^{2c}\cdot\frac{x+1}{x-1}$

∴ $\frac{y-1}{y+1}=e^{c}\cdot\frac{x+1}{x-1}$ ∴ $\frac{y-1}{y+1}=c\cdot\frac{x+1}{x-1}$ ∴ $(y-1)(x-1)=c(x+1)(y+1)$

∴ $xy-y-x+1=c(xy+x+y+1)=cxy+cx+cy+c$

∴ $xy-y-cxy-cy=cx+c+x-1$ ∴ $\{(1-c)x-(1+c)\}y=(c+1)x+(c-1)$

∴ $y=\frac{(1+c)x-(1-c)}{(1-c)x-(1+c)}=\frac{x-\frac{1-c}{1+c}}{\frac{1-c}{1+c}x-1}$ ∴ $y=\frac{x-c}{cx-1}$

(8) $\frac{dy}{dx}=-\frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$ ∴ $\frac{dy}{\sqrt{1-y^2}}=-\frac{dx}{\sqrt{1-x^2}}$ ∴ $\int\frac{dy}{\sqrt{1-y^2}}=-\int\frac{dx}{\sqrt{1-x^2}}+c$

∴ $\sin^{-1}y+\sin^{-1}x=c$. ここで $u=\sin^{-1}x$, $v=\sin^{-1}y$ とおくと. $x=\sin u$, $y=\sin v$.

∴ $\sin(u+v)=\sin u\cos v+\cos u\sin v=x\sqrt{1-y^2}+y\sqrt{1-x^2}$

∴ $x\sqrt{1-y^2}+y\sqrt{1-x^2}=\sin c$ ∴ $x\sqrt{1-y^2}+y\sqrt{1-x^2}=c$

$\boxed{2}$ (1) $u=y-x$ とおくと $\frac{du}{dx}=y'-1$ ∴ $1+\frac{du}{dx}=u^2$ ∴ $\frac{du}{u^2-1}=dx$

∴ $\frac{1}{2}\log\frac{u-1}{u+1}=x+c$ ∴ $\log\frac{u-1}{u+1}=2x+2c=\log e^{2c}\cdot e^{2x}$ ∴ $\frac{u-1}{u+1}=e^{2c}\cdot e^{2x}$

∴ $\frac{u-1}{u+1}=c_1e^{2x}$ ∴ $\frac{y-x-1}{y-x+1}=c_1e^{2x}$ ∴ $y-x-1=c_1e^{2x}y-c_1xe^{2x}+c_1e^{2x}$

∴ $(1-c_1e^{2x})y=x+1-c_1xe^{2x}+c_1e^{2x}$

∴ $(1-c_1e^{2x})y=x(1-c_1e^{2x})+1+c_1e^{2x}$ ∴ $y=x+\frac{1+c_1e^{2x}}{1-c_1e^{2x}}$

∴ $y=x+\frac{1+ce^{2x}}{1-ce^{2x}}$

(2) $u = x + e^{y}$ とおくと $\frac{du}{dx} = 1 + e^{y} \cdot y'$ $\therefore \frac{du}{dx} - 1 = u - 1$ $\therefore \frac{du}{dx} = u$ $\therefore \frac{du}{u} = dx$

$\therefore \log u = x + c = \log e^{c}\, e^{x}$ $\therefore u = e^{c} \cdot e^{x}$ $\therefore u = c \cdot e^{x}$ $\therefore x + e^{y} = c\, e^{x}$

$\therefore e^{y} = c e^{x} - x$ $\therefore \underline{y = \log (c e^{x} - x)}$

(3) $u = xy$ とおくと $\frac{du}{dx} = y + xy'$ $\therefore xy' = \frac{du}{dx} - y$

$\therefore y' = \frac{1}{x}\frac{du}{dx} - \frac{y}{x} = \frac{1}{x}\frac{du}{dx} - \frac{u}{x^2}$

$\therefore (1-u)\left\{\frac{1}{x}\frac{du}{dx} - \frac{u}{x^2}\right\} = y^2$ $\therefore (1-u)\frac{1}{x}\frac{du}{dx} - \frac{u(1-u)}{x^2} = y^2$

$\therefore x(1-u)\frac{du}{dx} - u(1-u) = u^2$ $\therefore x(1-u)\frac{du}{dx} = u$ $\therefore \left(\frac{1}{u} - 1\right)du = \frac{dx}{x}$

$\therefore \log u - u = \log x + c$ $\therefore \log \frac{u}{x} = u + c$ $\therefore \log y = xy + c$ $\therefore y = e^{xy+c} = e^{c} \cdot e^{xy}$

$\therefore \underline{y = c e^{xy}}$

(4) $u = x - y$ とおくと $\frac{du}{dx} = 1 - y'$ $\therefore -\frac{du}{dx} = u \tan x$ $\therefore \frac{du}{u} = -\tan x\, dx$

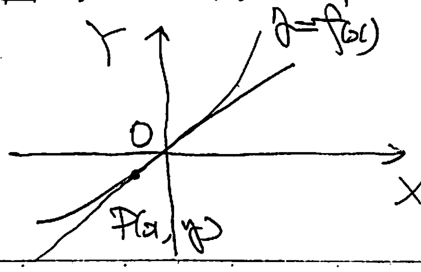
$\therefore \frac{du}{u} = \frac{-\sin x}{\cos x} dx$ $\therefore \log u = \log \cos x + c = \log e^{c} \cdot \cos x$ $\therefore u = e^{c} \cdot \cos x$

$\therefore u = c \cdot \cos x$ $\therefore x - y = c \cos x$ $\therefore \underline{y = x - c \cos x}$

$\boxed{3}$ $u = ax + by + c$ とおくと $\frac{du}{dx} = a + by'$ $\therefore y' = \frac{1}{b}\left(\frac{du}{dx} - a\right)$

$\therefore \frac{1}{b}\left(\frac{du}{dx} - a\right) = f(u)$ $\therefore \frac{du}{dx} = bf(u) + a$ ← xとuについての変数分離形

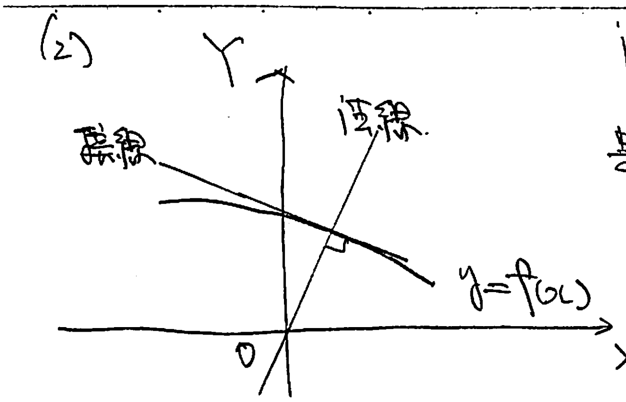
$\boxed{4}$ (1) 求める曲線を $y = f(x)$ とする 接線の方程式: $Y - y = \frac{dy}{dx}(X - x)$



原点を通る $0 - y = \frac{dy}{dx}(0 - x)$ $\therefore x\frac{dy}{dx} = y$ $\therefore \frac{dy}{y} = \frac{dx}{x}$

$\therefore \log y = \log x + c = \log e^{c} \cdot x$ $\therefore y = e^{c} \cdot x$ $\therefore \underline{y = cx}$

(2)



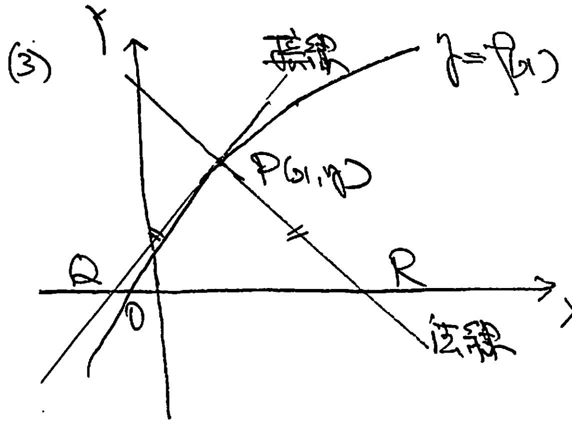
法線の方程式 $Y-y=-\dfrac{1}{\frac{dy}{dx}}(X-x)$

題意より $0-y=-\dfrac{1}{\frac{dy}{dx}}(0-x)$ ∴ $y\dfrac{dy}{dx}=-x$

∴ $ydy=-xdx$ ∴ $\dfrac{y^2}{2}=-\dfrac{x^2}{2}+c$

∴ $x^2+y^2=2c$ ∴ $x^2+y^2=c$

(3)



接線の方程式: $Y-y=\dfrac{dy}{dx}(X-x)$

Qの座標: $\left(x-\dfrac{y}{\frac{dy}{dx}},\ y\right)$

法線の方程式: $Y-y=-\dfrac{1}{\frac{dy}{dx}}(X-x)$

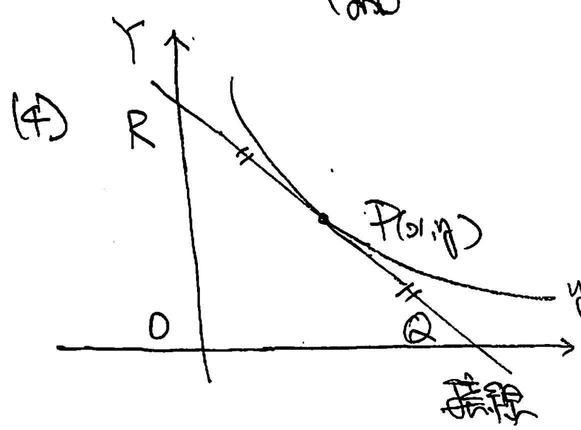
Rの座標 $\left(x+\dfrac{dy}{dx}y,\ y\right)$

∴ $\overline{PQ}^2=\left\{x-\left(x-\dfrac{y}{\frac{dy}{dx}}\right)\right\}^2+y^2=\dfrac{y^2}{\left(\frac{dy}{dx}\right)^2}+y^2$

$\overline{PR}^2=\left\{x-\left(x+\dfrac{dy}{dx}\cdot y\right)\right\}^2+y^2=\left(\dfrac{dy}{dx}\right)^2y^2+y^2$

$\overline{PQ}^2=\overline{PR}^2$ より $\dfrac{y^2}{\left(\frac{dy}{dx}\right)^2}=\left(\dfrac{dy}{dx}\right)^2y^2$ ∴ $\left(\dfrac{dy}{dx}\right)^4=1$ ∴ $\dfrac{dy}{dx}=\pm 1$ ∴ $y=\pm x+c$

(4)



接線の方程式 $Y-y=\dfrac{dy}{dx}(X-x)$

Qの座標 $\left(x-\dfrac{y}{\frac{dy}{dx}},\ 0\right)$, Rの座標 $\left(0,\ y-x\dfrac{dy}{dx}\right)$

題意より $x=\dfrac{x-\frac{y}{\frac{dy}{dx}}}{2}$ かつ $y=\dfrac{y-x\frac{dy}{dx}}{2}$

∴ $y=-x\dfrac{dy}{dx}$ ∴ $\dfrac{dy}{y}=-\dfrac{dx}{x}$

∴ $\log y=-\log x+c=\log\dfrac{e^c}{x}$ ∴ $xy=e^c$ ∴ $xy=c$