問題1.1

[1] (1) 1階常微分方程式 (2) 2階偏微分方程式 (3) 1階偏微分方程式
(4) 3階常微分方程式 (5) 2階常微分方程式 (6) 4階偏微分方程式

[2] (1) $y = C\cos x \quad \therefore y' = -C\sin x$. よって $\dfrac{y'}{y} = \dfrac{-C\sin x}{C\cos x} = -\tan x \quad \therefore y' + y\tan x = 0$.

$1 = y(0) = C\cdot\cos 0 = C \quad \therefore C = 1 \quad \therefore y = \cos x$

(2) $\sin y = x - 1 + Ce^{-x}$ の両辺を x で微分すると. $y'\cos y = 1 - Ce^{-x}$. よって.

$\sin y = x - (1 - Ce^{-x}) = x - y'\cos y \quad \therefore y'\cos y + \sin y = x \quad \therefore y' + \tan y = \dfrac{x}{\cos y}$.

$x = 0$ のとき $y = \dfrac{\pi}{6}$ であるから. $\sin\dfrac{\pi}{6} = 0 - 1 + C \quad \therefore C = 1 + \dfrac{1}{2} = \dfrac{3}{2} \quad \therefore \sin y = x - 1 + \dfrac{3}{2}e^{-x}$

(3) $\dfrac{dx}{dt} = -\dfrac{2C(t-1)}{(t-1)^4} = -\dfrac{2C}{(t-1)^3}$, $\dfrac{dy}{dt} = \dfrac{2Ct(t-1)^2 - Ct^2\cdot 2(t-1)}{(t-1)^4}$

$= \dfrac{2Ct(t-1) - 2Ct^2}{(t-1)^3} = \dfrac{2Ct^2 - 2Ct - 2Ct^2}{(t-1)^3} = \dfrac{-2Ct}{(t-1)^3}$

$\therefore y' = \dfrac{dy}{dt}\Big/\dfrac{dx}{dt} = \dfrac{-2Ct}{(t-1)^3}\cdot\left(-\dfrac{(t-1)^3}{2C}\right) = t \quad \therefore x(y')^2 - (y')^2 = xt^2 - t^2 = t^2(x-1)$

$= t^2\cdot\dfrac{C}{(t-1)^2} = y \quad \therefore y = x(y')^2 - (y')^2$.

$x = 1 + \dfrac{C}{(t-1)^2}$

$x = 0$ のとき $y = 0$ とする. このとき. $0 = 1 + \dfrac{C}{(t-1)^2} \quad \therefore C = -(t-1)^2$.

$\therefore y = \dfrac{t^2}{(t-1)^2}\left(-(t-1)^2\right) = -t^2$. $y = 0$ であるから $t = 0 \quad \therefore C = -1$. よって

$$\begin{cases} x = 1 - \dfrac{1}{(t-1)^2} \\ y = \dfrac{-t^2}{(t-1)^2} \end{cases}$$

(4) $y = C_1e^x + C_2e^{-x}$. $y' = C_1e^x - C_2e^{-x}$. $y'' = C_1e^x + C_2e^{-x} = y \quad \therefore y'' - y = 0$

$y(0) = C_1 + C_2 = 0$, $y(1) = C_1e + C_2e^{-1} = 1 \quad \therefore C_2 = -C_1$. $\therefore \dfrac{C_1e^2 - C_1}{e} = 1$

$c_1 = \dfrac{e}{e^2-1}$ $\therefore c_2 = -\dfrac{e}{e^2-1}$ $\therefore \underline{y = \dfrac{e}{e^2-1}\left(e^{x} - e^{-x}\right)}$

$\boxed{3}$ (1) $cy = x + \dfrac{c^2}{2}$ $\therefore cy' = 1$ $\therefore c = \dfrac{1}{y'}$ $\therefore \dfrac{y}{y'} = x + \dfrac{1}{2(y')^2}$

$\therefore \underline{2yy' = 2x(y')^2 + 1}$

(2) $\tan(x+c) = \dfrac{y}{x}$ $\therefore x + c = \tan^{-1}\dfrac{y}{x}$. 両辺をxで微分すると

$1 = \dfrac{1}{1+\left(\frac{y}{x}\right)^2} \cdot \dfrac{y'x - y}{x^2} = \dfrac{xy'-y}{x^2+y^2}$ $\therefore \underline{xy' - y = x^2 + y^2}$

(3) $y = x^c$ $\therefore \log y = c \log x$ $\therefore c = \dfrac{\log y}{\log x}$. 両辺をxで微分すると

$0 = \dfrac{\frac{1}{y} y' \log x - (\log y)\frac{1}{x}}{(\log x)^2}$ $\therefore \dfrac{y'}{y}\log x = \dfrac{\log y}{x}$ $\therefore \underline{y' \cdot x \log x = y \log y}$

(4) $y = c_1 e^{x} + c_2 x e^{2x}$

$y' = c_1 e^{x} + c_2 e^{2x} + 2c_2 x e^{2x}$

$\therefore y' - y = c_2 e^{2x} + c_2 x e^{2x}$ $\therefore c_2(1+x) = e^{-2x}(y'-y)$ $\cdots$ ①

①の両辺をxで微分:

$c_2 = -2e^{-2x}(y'-y) + e^{-2x}(y''-y') = e^{-2x}(-2y'+2y+y''-y') = e^{-2x}(y''-3y'+2y)$

これを①に代入

$(1+x)\cancel{e^{-2x}}(y''-3y'+2y) = \cancel{e^{-2x}}(y'-y)$ $\therefore y''-3y'+2y+xy''-3xy'+2xy-y'+y=0$

$\therefore \underline{(1+x)y'' - (3x+4)y' + (2x+3)y = 0}$

(5) $y = \dfrac{1}{c_1x+c_2} + 1$ $\therefore y - 1 = \dfrac{1}{c_1x+c_2}$ $\therefore y' = -\dfrac{c_1}{(c_1x+c_2)^2} = -c_1(y-1)^2$

$\therefore \dfrac{y'}{(y-1)^2} = -c_1$ 両辺をxで微分:

$$\frac{y''(y-1)^2-y'\cdot 2(y-1)y'}{(y-1)^4}=0 \quad \therefore y''(y-1)^2-2(y')^2(y-1)=0$$

$$\therefore (y-1)y''-2(y')^2=0$$

(6) $C_1x^2+C_2y^2=1 \quad \therefore 2C_1x+2C_2y\cdot y'=0 \cdots ①$

$\therefore 2C_1+2C_2\{(y')^2+y\cdot y''\}=0 \cdots ②$

②×x : $2C_1x+2C_2\{x(y')^2+xy\cdot y''\}=0 \cdots ②'$

②'−① : $2C_2\{x(y')^2+xy\cdot y''\}-2C_2yy'=0 \quad \therefore x(y')^2+xyy''=yy'$