問題1.3.2

$\boxed{1}$ (1) $\lambda^2-2\lambda-8=(\lambda-4)(\lambda+2)=0$ $\therefore \lambda=-2,4$ $\therefore y=C_1e^{-2x}+C_2e^{4x}$

(2) $\lambda^2-6\lambda+9=(\lambda-3)^2=0$ $\therefore \lambda=3$ (重解) $\therefore y=(C_1+C_2x)e^{3x}$

(3) $\lambda^2+\lambda=\lambda(\lambda+1)=0$ $\therefore \lambda=-1,0$ $\therefore y=C_1e^{0\cdot x}+C_2e^{-x}$ $\therefore y=C_1+C_2e^{-x}$

(4) $\lambda^2-2\lambda+2=(\lambda-1)^2+1=0$ $\therefore \lambda=1\pm i$ $\therefore y=e^{x}(C_1\sin x+C_2\cos x)$

(5) $\lambda^2+3=0$ $\therefore \lambda=\pm\sqrt{3}i$ $\therefore y=e^{0}\cdot(C_1\sin\sqrt{3}x+C_2\cos\sqrt{3}x)$ $\therefore y=C_1\sin\sqrt{3}x+C_2\cos\sqrt{3}x$

(6) $\lambda^2-4\lambda+6=(\lambda-2)^2+2=0$ $\therefore \lambda=2\pm\sqrt{2}i$ $\therefore y=e^{2x}(C_1\sin\sqrt{2}x+C_2\cos\sqrt{2}x)$

$\boxed{2}$ (1) $u=\log x$ とおくと $\frac{dy}{dx}=\frac{dy}{du}\cdot\frac{du}{dx}=\frac{1}{x}\cdot\frac{dy}{du}$, $\frac{d^2y}{dx^2}=\frac{d}{dx}\left(\frac{1}{x}\frac{dy}{du}\right)=-\frac{1}{x^2}\frac{dy}{du}$
$+\frac{1}{x}\frac{d}{dx}\left(\frac{dy}{du}\right)=-\frac{1}{x^2}\frac{dy}{du}+\frac{1}{x}\left\{\frac{d}{du}\left(\frac{dy}{du}\right)\cdot\frac{du}{dx}\right\}=-\frac{1}{x^2}\frac{dy}{du}+\frac{1}{x^2}\frac{d^2y}{du^2}$. これらを与式に代入:

$x^2\left(-\frac{1}{x^2}\frac{dy}{du}+\frac{1}{x^2}\frac{d^2y}{du^2}\right)-x\cdot\frac{1}{x}\frac{dy}{du}-3y=0$ $\therefore \frac{d^2y}{du^2}-2\frac{dy}{du}-3y=0$.

$\lambda^2-2\lambda-3=(\lambda-3)(\lambda+1)=0$ $\therefore \lambda=-1,3$. よって $y=C_1e^{-u}+C_2e^{3u}$

$\therefore y=C_1e^{-\log x}+C_2e^{3\log x}=C_1/x+C_2x^3$

(2) $u=\log x$ とおく. (1)と同様の計算により, $\frac{d^2y}{du^2}-4\frac{dy}{du}+5y=0$ $\therefore \lambda^2-4\lambda+5=0$

$\therefore \lambda=2\pm i$ $\therefore y=C_1e^{2u}\sin u+C_2e^{2u}\cos u$. $\therefore y=C_1x^2\sin\log x+C_2x^2\cos\log x$.

(3) $u=\log x$ とおく. (1)と同様の計算より. $\frac{d^2y}{du^2}+4\frac{dy}{du}+4y=0$

$\therefore \lambda^2+4\lambda+4=(\lambda+2)^2=0$ $\therefore \lambda=-2$ (重解) $\therefore y=(C_1+C_2u)e^{-2u}$

$\therefore y=(C_1+C_2\log x)e^{-2\log x}=(C_1+C_2\log x)/x^2$

(4) $y=\log x$ とおく. (1)と同様の計算により $\dfrac{d^2y}{du^2}+6\dfrac{dy}{du}+11y=0$

$\therefore \lambda^2+6\lambda+11=(\lambda+3)^2+2=0 \quad \therefore \lambda=-3\pm\sqrt{2}i \quad \therefore y=e^{-3u}(C_1\sin\sqrt{2}u+C_2\cos\sqrt{2}u)$

$\therefore y=e^{-3\log x}\left(C_1\sin(\sqrt{2}\log x)+C_2\cos(\sqrt{2}\log x)\right)=\underline{\left(C_1\sin(\sqrt{2}\log x)+C_2\cos(\sqrt{2}\log x)\right)/x^3}$