## The Possibility of Efficient Provision of a Public Good in Voluntary Participation Games<sup>\*</sup>

Ryusuke Shinohara $^{\dagger}$ 

Faculty of Economics, Shinshu University

## Abstract

In this study, we examine the allocative efficiency of Nash equilibria in a voluntary participation game in which a public good is provided in units of nonnegative integers. We show that the participation game has a Nash equilibrium that supports an efficient allocation and that some Nash equilibria are strong equilibria if at most one unit of the public good can be provided. However, the Nash equilibria of the participation game do not necessarily support efficient allocations if up to two units of the public good can be provided. We investigate the possibility of attaining efficient allocations at Nash equilibria in the case in which at most two units of the public good can be produced. We prove that Nash equilibria are less likely to support efficient allocations if the participation of many agents is needed for the efficient provision of the public good in the case of identical agents.

**Keywords:** Participation game, Nash equilibrium, Efficiency, Public project, Multi-unit public good, Diminishing rate of marginal benefits.

<sup>\*</sup> I would like to thank Koichi Tadenuma, Yukihiro Nishimura, Tatsuyoshi Saijo, Takehiko Yamato, Toshiji Miyakawa, Takashi Shimizu, Nobue Suzuki, and Dirk T. G. Rübbelke for helpful comments and suggestions. I am also grateful to participants of the 61th Congress of the International Institute of Public Finance (ICC Jeju, Korea; August 2005) and the 62th Congress of the Japan Institute of Public Finance (Hitotsubashi University; October 2005) for helpful comments. This research was supported by the Japanese Economic Research Foundation and the Grand-in-Aid for Young Scientists (Start-up) from the Japan Society for the Promotion of Science. Any remaining errors are my own.

<sup>&</sup>lt;sup>†</sup> *E-mail*: ryushinohara@yahoo.co.jp; *Postal Addres*: Faculty of Economics, Shinshu University, 3-1-1, Asahi, Matsumoto, Nagano, 390-8621, JAPAN.